積層膜からの和周波発生(SFG) — 表式

準備

原点：光線の入射点に取る。（広がりがある場合にはその中央）

z 軸：媒質１から媒質２に向けた法線。

x 軸：界面上でみた光の進行方向にとる。（k ベクトルの界面への射影に沿わせる）

次のように用語を定義する。

1/m 界面：媒質-1 と膜の界面。

m/2 界面：媒質-1 と膜の界面。

n_1, n_2, n_m：媒質-1、媒質-2、膜 m の屈折率。

k_1, k_2, k_m：媒質-1、媒質-2、膜 m の中での波動ベクトル。

$\theta_1, \theta_2, \theta_m$：媒質-1、媒質-2、膜 m の中を進む光の光路が法線に対してなす角。

t_{1m}：媒質-1 側から膜側に透過する光の電場に対する透過係数。

t_{m1}：膜側から媒質-1 側に透過する光の電場に対する透過係数。

r_{1m}：媒質-1 側から来て 1/m 界面で反射する光の電場に対する反射係数。

r_{m1}：膜側から来て m/1 界面で反射する光の電場に対する反射係数。

t_{m2}：膜側から媒質-2 側に透過する光の電場に対する透過係数。

t_{2m}：媒質-2 側から膜側に透過する光の電場に対する透過係数。

r_{m2}：膜側から来て m/2 界面で反射する光の電場に対する反射係数。

r_{2m}：媒質-2 側から来て 1/m 界面で反射する光の電場に対する反射係数。

上向き(-）光：m/2 界面から m/1 界面に向かう光。反射光と同じく、-z 側に進む。

下向き(+)光：m/1 界面から m/2 界面に向かう光。入射光と同じく、+z 側に進む。

（反射係数）

$$r_{12}^{s} = E^{s}_{r}/E^{s}_{i} = (k_{1z} - k_{2z})/(k_{1z} + k_{2z}) = (n_1\cos\theta_1 - n_2\cos\theta_2)/(n_1\cos\theta_1 + n_2\cos\theta_2)$$
$$= -\sin(\theta_1 - \theta_2)/\sin(\theta_1 + \theta_2)$$
$$r_{12}^{p} = E^{p}_{r}/E^{p}_{i} = (\varepsilon_1 k_{2z} - \varepsilon_2 k_{1z})/(\varepsilon_2 k_{1z} + \varepsilon_1 k_{2z}) = (n_1\cos\theta_2 - n_2\cos\theta_1)/(n_2\cos\theta_1 + n_1\cos\theta_2)$$
$$= -\tan(\theta_1 - \theta_2)/\tan(\theta_1 + \theta_2)$$

（透過係数）

$$t_{12}^{s} = E^{s}_{t}/E^{s}_{i} = 2k_{1z}/(k_{1z} + k_{2z}) = 2n_1\cos\theta_1/(n_1\cos\theta_1 + n_2\cos\theta_2)$$
$$= 2\cos\theta_1\sin\theta_2/\sin(\theta_1 + \theta_2)$$
$$t_{12}^{p} = E^{p}_{t}/E^{p}_{i} = 2\varepsilon_1 k_{2z}\cos\theta_1/(\varepsilon_2 k_{1z} + \varepsilon_1 k_{2z}) = 2n_1\cos\theta_1/(n_2\cos\theta_1 + n_1\cos\theta_2)$$
$$= 2\cos\theta_1\sin\theta_2/\sin(\theta_1 + \theta_2)\cos(\theta_1 - \theta_2)$$
$$r_{21}^{s} = -r_{12}^{s}\ ,\ r_{21}^{p} = -r_{12}^{p}$$
$$t_{21}^{s} = (n_2\cos\theta_2/n_1\cos\theta_1)t_{12}^{s}\ ,\ t_{21}^{p} = (n_2\cos\theta_2/n_1\cos\theta_1)t_{12}^{p}$$

界面の内向き法線をｚ 軸に、s 偏光の電場の方向をｙ 軸に、光の進行方向が正になるようにｘ 軸を定義して、電場の軸方向成分に対する係数を求めると、次のようになる。

$E_{x,i} = E^p{}_i\cos\theta_1$,　$E_{y,i} = E^s{}_{ri}$,　$E_{z,i} = -E^p{}_i\sin\theta_1$
$E_{x,r} = E^p{}_r\cos\theta_1$,　$E_{y,r} = E^s{}_r$,　$E_{z,r} = \mathrm{E}^p{}_r\sin\theta_1$
$E_{x,t} = E^p{}_t\cos\theta_2$,　$E_{y,t} = E^s{}_t$,　$E_{z,t} = -E^p{}_t\sin\theta_2$

上の表式より、下の振幅反射係数および振幅等価係数の表式を得る。

$r_{12}{}^x = r_{12}{}^p$,　$r_{12}{}^y = r_{12}{}^s$,　$r_{12}{}^z = -r_{12}{}^p$
$\mathrm{r}_{21}{}^x = -r_{12}{}^p$,　$r_{21}{}^y = -r_{12}{}^s$,　$r_{21}{}^z = r_{12}{}^p$
$t_{12}{}^x = (\cos\theta_2/\cos\theta_1)t_{12}{}^p$,　$t_{12}{}^y = t_{12}{}^s$,　$t_{12}{}^z = (\sin\theta_2/\sin\theta_1)t_{12}{}^p = (n_1/n_2)\, t_{12}{}^p$
$\mathrm{t}_{21}{}^x = (n_2/n_1)t_{12}{}^p$,　$t_{12}{}^y = t_{12}{}^s$,　$t_{12}{}^z = (\cos\theta_2/\cos\theta_1)\,(n_2/n_1)^2 t_{12}{}^p$

$r_{1m,x} = -r_{m1,x} = r_{1m,p}$,　$r_{1m,y} = -r_{m1,y} = r_{1m,s}$,　$r_{1m,z} = -r_{m1,z} = -r_{1m,p}$
$r_{2m,x} = -r_{m2,x} = r_{2m,p}$,　$r_{2m,y} = -r_{m2,y} = r_{2m,s}$,　$r_{2m,z} = -r_{m2,z} = -r_{2m,p}$
$t_{1m,x} = (\cos\theta_m/\cos\theta_1)t_{1m,p}$,　$t_{1m,y} = t_{1m,s}$,　$t_{1m,z} = (\sin\theta_m/\sin\theta_1)t_{1m,p}$
$t_{m1,x} = (\cos\theta_1/\cos\theta_m)t_{m1,p}$,　$t_{m1,y} = t_{m1,s}$,　$t_{m1,z} = (\sin\theta_1/\sin\theta_m)t_{m1,p}$
$t_{2m,x} = (\cos\theta_m/\cos\theta_2)t_{2m,p}$,　$t_{2m,y} = t_{2m,s}$,　$t_{2m,z} = (\sin\theta_m/\sin\theta_2)t_{2m,p}$
$t_{m2,x} = (\cos\theta_2/\cos\theta_m)t_{m2,p}$,　$t_{m2,y} = t_{m2,s}$,　$t_{m2,z} = (\sin\theta_2/\sin\theta_m)t_{m2,p}$
$t_{1m,\alpha}t_{m1,\alpha} = 1 + r_{1m,\alpha}r_{m1,\alpha} = 1 - r_{1m,\alpha}{}^2$,　$t_{2m,\alpha}t_{m2,\alpha} = 1 + r_{2m,\alpha}r_{m2,\alpha} = 1 - r_{2m,\alpha}{}^2$,　($\alpha$ = x, y, z)

上式は次のような関係式を満足する。

$1 + r_{12}{}^x = t_{12}{}^x$,　$1 + r_{12}{}^y = \mathrm{t}_{12}{}^y$,　$1 + r_{12}{}^z = (n_2/n_1)^2 t_{12}{}^z$　**（電場に対する境界条件）**
$1 + r_{12}{}^s r_{21}{}^s = 1 - (r_{12}{}^s)^2 = t_{12}{}^s\, t_{21}{}^s$,　$1 + r_{12}{}^p r_{21}{}^p = 1 - (r_{12}{}^p)^2 = t_{12}{}^p t_{21}{}^p$

因みに、光の強度は $E^2 n\cos\theta$ **に比例するので、**$r_{12}{}^2 n_1\cos\theta_1 + t_{12}{}^2 n_2\cos\theta_2 = n_1\cos\theta_1$ **となり、光の強度は保存される。**

L係数、電場振幅

$E_{SF,\,\alpha} = \sum_\beta L_{\alpha\beta} P_\beta{}^{SF}$

共通因子である $4\pi i\omega_{SF}/c$**（屈折率の代わりに波数ベクトルを使うときには** $4\pi i\omega_{SF}{}^2/c^2$**）を省略する。**

L係数の表記法；$L_{i/j,s(p),\alpha}$**： 分極シート m' が i 層とj 層に挟まれているときに、分極の α 成分** (α = x, y, z) **が作る光の s 偏光成分又は p 偏光成分の間の係数**（x、y、z **成分ではないことに注意）。上向き** (-) **光、上向き** (+) **光の区別を上付き -、+ で示す。**

媒質 1 と積層膜 m の間の分極シート m' からの光生成に対しては、

$L^-{}_{1/m,p,x} = \cos\theta_{m,SF}/(n_{1,SF}\cos\theta_{m,SF} + n_{m,SF}\cos\theta_{1,SF})$
$L^-{}_{1/m,s,y} = 1/(n_{1,SF}\cos\theta_{1,SF} + n_{m,SF}\cos\theta_{m,SF})$
$L^-{}_{1/m,p,z} = (n_{m,SF}/n_{m',SF})\sin\theta_{m',SF}/(n_{1,SF}\cos\theta_{m,SF} + n_{m,SF}\cos\theta_{1,SF})$

$= (n_{m,SF}/n_{m',SF})^2 \sin\theta_{m,SF}/(n_{1,SF}\cos\theta_{m,SF} + n_{m,SF}\cos\theta_{1,SF})$

$L^+_{1/m,p,x} = \cos\theta_{1,SF}/(n_{1,SF}\cos\theta_{m,SF} + n_{m,SF}\cos\theta_{1,SF})$

$L^+_{1/m,s,y} = 1/(n_{1,SF}\cos\theta_{1,SF} + n_{m,SF}\cos\theta_{m,SF})$

$L^+_{1/m,p,z} = -(n_{1,SF}/n_{m',SF})\sin\theta_{m',SF}/(n_{1,SF}\cos\theta_{m,SF} + n_{m,SF}\cos\theta_{1,SF})$

$= -(n_{1,SF}/n_{m',SF})^2 \sin\theta_{1,SF}/(n_{1,SF}\cos\theta_{m,SF} + n_{m,SF}\cos\theta_{1,SF})$

媒質２と積層膜 m の間の分極シート m'' からの光生成に対しては、

$L^-_{2/m,p,x} = \cos\theta_{2,SF}/(n_{2,SF}\cos\theta_{m,SF} + n_{m,SF}\cos\theta_{2,SF})$

$L^-_{2/m,s,y} = 1/(n_{2,SF}\cos\theta_{2,SF} + n_{m,SF}\cos\theta_{m,SF})$

$L^-_{2/m,p,z} = (n_{2,SF}/n_{m'',SF})\sin\theta_{m'',SF}/(n_{2,SF}\cos\theta_{m,SF} + n_{m,SF}\cos\theta_{2,SF})$

$= (n_{2,SF}/n_{m'',SF})^2 \sin\theta_{2,SF}/(n_{2,SF}\cos\theta_{m,SF} + n_{m,SF}\cos\theta_{2,SF})$

$L^+_{2/m,p,x} = \cos\theta_{m,SF}/(n_{2,SF}\cos\theta_{m,SF} + n_{m,SF}\cos\theta_{2,SF})$

$L^+_{2/m,s,y} = 1/(n_{2,SF}\cos\theta_{2,SF} + n_{m,SF}\cos\theta_{m,SF})$

$L^+_{2/m,p,z} = -(n_{m,SF}/n_{m'',SF})\sin\theta_{m'',SF}/(n_{2,SF}\cos\theta_{m,SF} + n_{m,SF}\cos\theta_{2,SF})$

$= -(n_{m,SF}/n_{m'',SF})^2 \sin\theta_{m,SF}/(n_{2,SF}\cos\theta_{m,SF} + n_{m,SF}\cos\theta_{2,SF})$

積層膜内部の分極シートからの光生成に対しては、

$L^-_{m/m,p,x} = L^+_{m/m,p,x} = \cos\theta_{m,SF}/(2n_{m,SF}\cos\theta_{m,SF})$

$L^-_{m/m,s,y} = L^+_{m/m,s,y} = 1/(2n_{m,SF}\cos\theta_{m,SF})$

$L^-_{m/m,p,z} = -L^+_{m/m,p,z} = \sin\theta_{m,SF}/(2n_{m,SF}\cos\theta_{m,SF})$

よって、各部位からの SFG **光の電場振幅は次のように表される。**

$\boldsymbol{P}(0)$ に由来するもの：（$\boldsymbol{P}(0^-)$ は媒質１側の電場による分極､、$\boldsymbol{P}(0^+)$ は膜側の電場による分極）

$E^-_{1,p}(z = 0^-) = L^-_{1/m,p,x}P_x(0^-) + L^-_{1/m,p,z}P_z(0^-)$

$E^-_{1,s}(z = 0^-) = L^-_{1/m,s,y}P_y(0^-)$

$E^+_{m,p}(z = 0^+) = L^+_{1/m,p,x}P_x(0^+) + L^+_{1/m,p,z}P_z(0^+)$

$E^+_{m,s}(z = 0^+) = L^+_{1/m,s,y}P_y(00^+)$

$\boldsymbol{P}(h_m)$ に由来するもの：（$\boldsymbol{P}(h_m^+)$ は媒質２側の電場による分極､、$\boldsymbol{P}(h_m^-)$ は膜側の電場による分極）

$E^-_{m,p}(z = h_m^-) = L^-_{2/m,p,x}P_x(h_m^-) + L^-_{2/m,p,z}P_z(h_m^-)$

$E^-_{m,s}(z = h_m^-) = L^-_{2/m,s,y}P_y(h_m^-)$

$E^+_{2,p}(z = h_m^+) = L^+_{2/m,p,x}P_x(h_m^+) + L^+_{2/m,p,z}P_z(h_m^+)$

$E^+_{2,s}(z = h_m^+) = L^+_{2/m,s,y}P_y(h_m^+)$

$\boldsymbol{P}(z)$ に由来するもの：

$E^-_{m,p}(z) = L^-_{m/m,p,x}P_x(z) + L^-_{m/m,p,z}P_z(z)$

$E^-_{m,s}(z) = L^-_{m/m,s,y}P_y(z)$

$E^{+}{}_{\mathrm{m,p}}(z) = L^{+}{}_{\mathrm{m/m,p,x}}P_{\mathrm{x}}(z) + L^{+}{}_{\mathrm{m/m,p,z}}P_{\mathrm{z}}(z)$

$E^{+}{}_{\mathrm{m,s}}(z) = L^{+}{}_{\mathrm{m/m,s,y}}P_{\mathrm{y}}(z)$

1/m 界面の外側 (x = -2nh_{m}tanθ$_{\mathrm{m,SF}}$, z = 0^{-}) からの SFG

反射 SFG

$$E_{1p}(0^-) = \sum_{\beta\gamma} E^0{}_{vis,\beta} E^0{}_{ir,\gamma} \frac{(1+r_{1m,vis,\beta})(1+r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})(1+r_{1m,ir,\gamma})(1+r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})}{(1+r_{1m,vis,\beta}r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})(1+r_{1m,ir,\gamma}r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})} \times \frac{(1+r_{m2,SF,p}e^{2i\beta_{m,SF}h_m})L^{-}{}_{1/m,px}\chi_{x\beta\gamma} + (1-r_{m2,SF,p}e^{2i\beta_{m,SF}h_m})L^{-}{}_{1/m,pz}\chi_{z\beta\gamma}}{(1+r_{1m,SF,p}r_{m2,SF,p}e^{2i\beta_{m,SF}h_m})} \tag{1a}$$

$$E_{1s}(0^-) = \sum_{\beta\gamma} E^0{}_{vis,\beta} E^0{}_{ir,\gamma} \frac{(1+r_{1m,vis,\beta})(1+r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})(1+r_{1m,ir,\gamma})(1+r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})}{(1+r_{1m,vis,\beta}r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})(1+r_{1m,ir,\gamma}r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})} \times \frac{(1+r_{m2,SF,s}e^{2i\beta_{m,SF}h_m})}{(1+r_{1m,SF,s}r_{m2,SF,s}e^{2i\beta_{m,SF}h_m})} L^{-}{}_{1/m,sy}\chi_{y\beta\gamma} \tag{1b}$$

透過 SFG

$$E^{+}{}_{2p}(h_m{}^{+}) = \sum_{\beta\gamma} E^0{}_{vis,\beta} E^0{}_{ir,\gamma} \frac{(1+r_{1m,vis,\beta})(1+r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})(1+r_{1m,ir,\gamma})(1+r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})}{(1+r_{1m,vis,\beta}r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})(1+r_{1m,ir,\gamma}r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})} \times \frac{t_{1m,SF,p}t_{m2,SF,p}e^{i\beta_{m,SF}h_m}}{1+r_{1m,SF,p}r_{m2,SF,p}e^{2i\beta_{m,SF}h_m}}[\chi_{x\beta\gamma}L^{+}{}_{m/1,px} + \chi_{z\beta\gamma}L^{+}{}_{m/1,pz}] \tag{2a}$$

$$E^{+}{}_{2s}(h_m{}^{+}) = \sum_{\beta,\gamma} E^0{}_{vis,\beta} E^0{}_{ir,\gamma} \frac{(1+r_{1m,vis,\beta})(1+r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})(1+r_{1m,ir,\gamma})(1+r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})}{(1+r_{1m,vis,\beta}r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})(1+r_{1m,ir,\gamma}r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})} \times \frac{t_{1m,SF,s}t_{m2,SF,s}e^{i\beta_{m,SF}h_m}}{1+r_{1m,SF,s}r_{m2,SF,s}e^{2i\beta_{m,SF}h_m}}\chi_{y\beta\gamma}L^{+}{}_{m/1,sy} \tag{2b}$$

表面が薄膜として完成しているときの薄膜（屈折率 m'）を形成する分子からの SFG は、内部反射を考慮する必要がある。

反射 SFG

$$E^{-}{}_{1p}(0^-) = \sum_{\beta,\gamma} E^0{}_{vis,\beta} E^0{}_{ir,\gamma} \frac{t_{1m',vis,\beta}(1+r_{m'm,vis,\beta})(1+r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})}{1+r_{1m',vis,\beta}r_{m'm,vis,\beta} + (r_{m'm,vis,\beta}+r_{1m',vis,\beta})r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m}} \times \frac{t_{1m',ir,\gamma}(1+r_{m'm,ir,\gamma})(1+r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})}{1+r_{1m',ir,\gamma}r_{m'm,ir,\gamma} + (r_{m'm,ir,\gamma}+r_{1m',ir,\gamma})r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m}} \times \frac{(1+r_{1m',SF,p}r_{m'm,SF,p})[(1+r_{m2,SF,p}e^{2i\beta_{m,SF}h_m})\chi_{x\beta\gamma}L^{-}{}_{1/m,px} + (1-r_{m2,SF,p}e^{2i\beta_{m,SF}h_m})\chi_{z\beta\gamma}L^{-}{}_{1/m,pz}]}{1+r_{1m',SF,p}r_{m'm,SF,p} + r_{m2,SF,p}(r_{1m',SF,p}+r_{m'm,SF,p})e^{2i\beta_{m,SF}h_m}} \tag{3a}$$

$$E^{-}{}_{1s}(0^{-})=\sum_{\beta\gamma}E^{0}{}_{vis,\beta}E^{0}{}_{ir,\gamma}\ \frac{t_{1m',vis,\beta}(1+r_{m'm,vis,\beta})(1+r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})}{1+r_{1m',vis,\beta}r_{m'm,vis,\beta}+(r_{m'm,vis,\beta}+r_{1m',vis,\beta})r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m}}$$
$$\times\frac{t_{1m',ir,\gamma}(1+r_{m'm,ir,\gamma})(1+r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})}{1+r_{1m',ir,\gamma}r_{m'm,ir,\gamma}+(r_{m'm,ir,\gamma}+r_{1m',ir,\gamma})r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m}}$$
$$\times\frac{(1+r_{1m'SF,s}r_{m'mSF,s})(1+r_{m2SF,s}e^{2i\beta_{m,SF}h_m})}{1+r_{1m'SF,s}r_{m'mSF,s}+r_{m2SF,s}(r_{1m'SF,s}+r_{m'mSF,s})e^{2i\beta_{m,SF}h_m}}\chi_{y\beta\gamma}L^{-}{}_{1/m,sy} \tag{3b}$$

透過 SFG

$$E^{+}{}_{2p}(h_m{}^{+})=\sum_{\beta\gamma}E^{0}{}_{vis,\beta}E^{0}{}_{ir,\gamma}\ \frac{t_{1m',vis,\beta}(1+r_{m'm,vis,\beta})(1+r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})}{1+r_{1m',vis,\beta}r_{m'm,vis,\beta}+(r_{m'm,vis,\beta}+r_{1m',vis,\beta})r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m}}$$
$$\times\frac{t_{1m',ir,\gamma}(1+r_{m'm,ir,\gamma})(1+r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})}{1+r_{1m',ir,\gamma}r_{m'm,ir,\gamma}+(r_{m'm,ir,\gamma}+r_{1m',ir,\gamma})r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m}}$$
$$\times\frac{t_{m2,SF,p}(1+r_{1m',SF,p}r_{m'm,SF,p})e^{i\beta_{m,SF}h_m}}{1+r_{1m',SF,p}r_{m'm,SF,p}+r_{m2,SF,p}(r_{mm',SF,p}+r_{m'1,SF,p})e^{2i\beta_{m,SF}h_m}}(L^{+}{}_{1/m,px}\chi_{x\beta\gamma}+L^{+}{}_{1/m,pz}\chi_{z\beta\gamma}) \tag{4a}$$

$$E^{+}{}_{2s}(h_m{}^{+})=\sum_{\beta,\gamma}E^{0}{}_{vis,\beta}E^{0}{}_{ir,\gamma}\ \frac{t_{1m',vis,\beta}(1+r_{m'm,vis,\beta})(1+r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})}{1+r_{1m',vis,\beta}r_{m'm,vis,\beta}+(r_{m'm,vis,\beta}+r_{1m',vis,\beta})r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m}}$$
$$\times\frac{t_{1m',ir,\gamma}(1+r_{m'm,ir,\gamma})(1+r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})}{1+r_{1m',ir,\gamma}r_{m'm,ir,\gamma}+(r_{m'm,ir,\gamma}+r_{1m',ir,\gamma})r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m}}$$
$$\times\frac{t_{m2,SF,s}(1+r_{1m',SF,s}r_{m'm,SF,s})e^{i\beta_{m,SF}h_m}}{1+r_{1m',SF,s}r_{m'm,SF,s}+r_{m2,SF,s}(r_{mm',SF,s}+r_{m'1,SF,s})e^{2i\beta_{m,SF}h_m}})L^{+}{}_{1/m,sy}\chi_{y\beta\gamma} \tag{4b}$$

1/m 界面の内側 (x = -2$nh_m\tan\theta_{m,SF}$, z = 0^+) からの SFG

反射 SFG

$$E^{-}{}_{1p}(0^{-})=\sum_{\beta,\gamma}E^{0}{}_{vis,\beta}E^{0}{}_{ir,\gamma}\ \frac{t_{1m,vis,\beta}t_{1m,ir,\gamma}(1+r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})(1+r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})}{(1+r_{1m,vis,\beta}r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})(1+r_{1m,ir,\gamma}r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})}$$
$$\times\frac{[(1+r_{m2,SF,p}e^{2i\beta_{m,SF}h_m})L^{-}{}_{1/m,px}\chi_{x\beta\gamma}+(1-r_{m2,SF,p}e^{2i\beta_{m,SF}h_m})L^{-}{}_{1/m,pz}\chi_{z\beta\gamma}]}{1+r_{1m,SF,p}r_{m2,SF,p}e^{2i\beta_{m,SF}h_m}} \tag{5a}$$

$$E^{-}{}_{1s}(0^{-})=\sum_{\beta\gamma}E^{0}{}_{vis,\beta}E^{0}{}_{ir,\gamma}\ \frac{t_{1m,vis,\beta}t_{1m,ir,\gamma}(1+r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})(1+r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})}{(1+r_{1m,vis,\beta}r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})(1+r_{1m,ir,\gamma}r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})}$$
$$\times\frac{(1+r_{m2,SF,s}e^{2i\beta_{m,SF}h_m})L^{-}{}_{1/m,sy}\chi_{y\beta\gamma}}{1+r_{1m,SF,s}r_{m2,SF,s}e^{2i\beta_{m,SF}h_m}} \tag{5b}$$

透過 SFG

$$E^{+}{}_{2p}(h_m{}^{+})=\sum_{\beta\gamma}E^{0}{}_{vis,\beta}E^{0}{}_{ir,\gamma}\ \frac{t_{1m,vis,\beta}t_{1m,ir,\gamma}(1+r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})(1+r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})}{(1+r_{1m,vis,\beta}r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})(1+r_{1m,ir,\gamma}r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})}$$
$$\times\frac{t_{m2,SF,p}e^{i\beta_{m,SF}h_m}}{1+r_{1m,SF,p}r_{m2,SF,p}e^{2i\beta_{m,SF}h_m}}(L^{+}{}_{1/m,px}\chi_{x\beta\gamma}+L^{+}{}_{1/m,pz}\chi_{z\beta\gamma}) \tag{6a}$$

$$E^{+}{}_{2s}(h_m{}^{+}) = \sum_{\beta,\gamma} E^{0}{}_{vis,\beta} E^{0}{}_{ir,\gamma} \frac{t_{1m,vis,\beta} t_{1m,ir,\gamma} (1 + r_{m2,vis,\beta} e^{2i\beta_{m,vis} h_m})(1 + r_{m2,ir,\gamma} e^{2i\beta_{m,ir} h_m})}{(1 + r_{1m,vis,\beta} r_{m2,vis,\beta} e^{2i\beta_{m,vis} h_m})(1 + r_{1m,ir,\gamma} r_{m2,ir,\gamma} e^{2i\beta_{m,ir} h_m})}$$
$$\times \frac{t_{m2,SF,s} e^{i\beta_{m,SF} h_m}}{1 + r_{1m,SF,s} r_{m2,SF,s} e^{2i\beta_{m,SF} h_m}} L^{+}{}_{1/m,sy} \chi_{y\beta\gamma} \qquad (6b)$$

m/2 界面の外側 ($x = -(2n + 1)h_m \tan\theta_{m,SF}$, $z = h_m{}^{+}$) からの SFG

反射 SFG

$$E^{-}{}_{1p}(0^{-}) = \sum_{\beta,\gamma} E^{0}{}_{vis,\beta} E^{0}{}_{ir,\gamma} \frac{t_{1m,vis,\beta} t_{m2,vis,\beta} t_{1m,ir,\gamma} t_{m2,ir,\gamma} e^{i(\beta_{m,ir} + \beta_{m,vis}) h_m}}{(1 + r_{1m,vis,\beta} r_{m2,wvis,\beta} e^{2i\beta_{m,vis} h_m})(1 + r_{1m,ir,\gamma} r_{m2,ir,\gamma} e^{2i\beta_{m,ir} h_m})}$$
$$\times \frac{t_{m1,p} e^{i\beta_{m,SF} h_m}}{1 + r_{1m.p} r_{m2,p} 2e^{2i\beta_{m,SF} h_m}} [L^{-}{}_{2/m,px} \chi_{x\beta\gamma} + L^{-}{}_{2/m,pz} \chi_{z\beta\gamma}] \qquad (7a)$$

$$E^{-}{}_{1s}(0^{-}) = \sum_{\beta,\gamma} E^{0}{}_{vis,\beta} E^{0}{}_{ir,\gamma} \frac{t_{1m,vis,\beta} t_{m2,vis,\beta} t_{1m,ir,\gamma} t_{m2,ir,\gamma} e^{i(\beta_{m,ir} + \beta_{m,vis}) h_m}}{(1 + r_{1m,vis,\beta} r_{m2,wvis,\beta} e^{2i\beta_{m,vis} h_m})(1 + r_{1m,ir,\gamma} r_{m2,ir,\gamma} e^{2i\beta_{m,ir} h_m})}$$
$$\times \frac{t_{m1,s} e^{i\beta_{m,SF} h_m}}{1 + r_{1m.s} r_{m2,s} 2e^{2i\beta_{m,SF} h_m}} L^{-}{}_{2/m,sy} \chi_{y\beta\gamma} \qquad (7b)$$

透過 SFG

$$E^{+}{}_{2p}(h_m{}^{+}) = \sum_{\beta,\gamma} E^{0}{}_{vis,\beta} E^{0}{}_{ir,\gamma} \frac{t_{1m,vis,\beta} t_{m2,vis,\beta} t_{1m,ir,\gamma} t_{m2,ir,\gamma} e^{i(\beta_{m,ir} + \beta_{m,vis}) h_m}}{(1 + r_{1m,vis,\beta} r_{m2,wvis,\beta} e^{2i\beta_{m,vis} h_m})(1 + r_{1m,ir,\gamma} r_{m2,ir,\gamma} e^{2i\beta_{m,ir} h_m})}$$
$$\times \frac{(1 - r_{1m,SF,p} e^{2i\beta_{m,SF} h_m}) L^{+}{}_{2/m,px} \chi_{x\beta\gamma} + (1 + r_{1m,SF,p} e^{2i\beta_{m,SF} h_m}) L^{+}{}_{2/m,pz} \chi_{z\beta\gamma}}{1 + r_{1m,SF,p} r_{m2,SF,p} e^{2i\beta_{m,SF} h_m}} \qquad (8a)$$

$$E^{+}{}_{2s}(h_m{}^{+}) = \sum_{\beta,\gamma} E^{0}{}_{vis,\beta} E^{0}{}_{ir,\gamma} \frac{t_{1m,vis,\beta} t_{m2,vis,\beta} t_{1m,ir,\gamma} t_{m2,ir,\gamma} e^{i(\beta_{m,ir} + \beta_{m,vis}) h_m}}{(1 + r_{1m,vis,\beta} r_{m2,wvis,\beta} e^{2i\beta_{m,vis} h_m})(1 + r_{1m,ir,\gamma} r_{m2,ir,\gamma} e^{2i\beta_{m,ir} h_m})}$$
$$\times \frac{(1 - r_{1m,SF,s} e^{2i\beta_{m,SF} h_m}) L^{+}{}_{2/m,sy} \chi_{y\beta\gamma}}{1 + r_{1m,SF,s} r_{m2,SF,s} e^{2i\beta_{m,SF} h_m}} \qquad (8b)$$

SFG を出す界面が屈折率 m'' の薄膜の時には、

反射 SFG

$$E^{-}{}_{1p}(0^{-}) = \sum_{\beta,\gamma} E^{0}{}_{vis,\beta} E^{0}{}_{ir,\gamma} \frac{t_{1m,vis,\beta} t_{mm'',vis,\beta} (1 + r_{m''2,vis,\beta}) e^{i\beta_{m,vis} h_m}}{(1 + r_{mm'',vis,\beta} r_{m''2,vis,\beta}) + r_{1m,vis,\beta} (r_{mm'',vis,\beta} + r_{m''2,vis,\beta}) e^{2i\beta_{m,vis} h_m}}$$
$$\times \frac{t_{1m,ir,\gamma} t_{mm'',ir,\gamma} (1 + r_{m''2,ir,\gamma}) e^{i\beta_{m,ir} h_m}}{(1 + r_{mm'',ir,\gamma} r_{m''2,ir,\gamma}) + r_{1m,ir,\gamma} (r_{mm'',ir,\gamma} + r_{m''2,ir,\gamma}) e^{2i\beta_{m,ir} h_m}}$$
$$\times \frac{t_{m1,SF,p} e^{i\beta_{m,SF} h_m} (1 + r_{mm'',SF,p} r_{m''2,SF,p})}{1 + r_{mm'',SF,p} r_{m''2,SF,p} + r_{1m,SF,p} (r_{mm'',SF,p} + r_{m''2,SF,p}) e^{2i\beta_{m,SF} h_m}} [L^{-}{}_{2/m,px} \chi_{x\beta\gamma} + L^{-}{}_{2/m,pz} \chi_{z\beta\gamma}] \qquad (9a)$$

$$E^{-}{}_{1s}(0^{-})=\sum_{\beta\gamma}E^{0}{}_{vis,\beta}E^{0}{}_{ir,\gamma}\ \frac{t_{1m,vis,\beta}t_{mm'',vis,\beta}(1+r_{m''2,vis,\beta})e^{i\beta_{m,vis}h_m}}{(1+r_{mm'',vis,\beta}r_{m''2,vis,\beta})+r_{1m,vis,\beta}(r_{mm'',vis,\beta}+r_{m''2,vis,\beta})e^{2i\beta_{m,vis}h_m}}$$
$$\times\frac{t_{1m,ir,\gamma}t_{mm'',ir,\gamma}(1+r_{m''2,ir,\gamma})e^{i\beta_{m,ir}h_m}}{(1+r_{mm'',ir,\gamma}r_{m''2,ir,\gamma})+r_{1m,ir,\gamma}(r_{mm'',ir,\gamma}+r_{m''2,ir,\gamma})e^{2i\beta_{m,ir}h_m}}$$
$$\times\frac{t_{m1,SF,s}e^{i\beta_{m,SF}h_m}(1+r_{mm'',SF,s}r_{m''2,SF,s})}{1+r_{mm'',SF,s}r_{m''2,SF,s}+r_{1m,SF,s}(r_{mm'',SF,s}+r_{m''2,SF,s})e^{2i\beta_{m,SF}h_m}}L^{-}{}_{2/m,sy}\chi_{y\beta\gamma}\tag{9b}$$

透過 SFG

$$E_{2p}(h_m{}^{+})=\sum_{\beta\gamma}E^{0}{}_{vis,\beta}E^{0}{}_{ir,\gamma}\ \frac{t_{1m,vis,\beta}t_{mm'',vis,\beta}(1+r_{m''2,vis,\beta})e^{i\beta_{m,vis}h_m}}{(1+r_{mm'',vis,\beta}r_{m''2,vis,\beta})+r_{1m,vis,\beta}(r_{mm'',vis,\beta}+r_{m''2,vis,\beta})e^{2i\beta_{m,vis}h_m}}$$
$$\times\frac{t_{1m,ir,\gamma}t_{mm'',ir,\gamma}(1+r_{m''2,ir,\gamma})e^{i\beta_{m,ir}h_m}}{(1+r_{mm'',ir,\gamma}r_{m''2,ir,\gamma})+r_{1m,ir,\gamma}(r_{mm'',ir,\gamma}+r_{m''2,ir,\gamma})e^{2i\beta_{m,ir}h_m}}$$
$$\times\frac{(1+r_{mm'',SF,p}r_{m''2,SF,p})[(1-r_{1m,SF,p}e^{2i\beta_{m,SF}h_m})L^{+}{}_{2/m,px}\chi_{x\beta\gamma}+(1+r_{1m,SF,p}e^{2i\beta_{m,SF}h_m})L^{+}{}_{2/m,pz}\chi_{z\beta\gamma}]}{1+r_{mm'',SF,p}r_{m''2,SF,p}+r_{1m,SF,p}(r_{mm'',SF,p}+r_{m''2,SF,p})e^{2i\beta_{m,SF}h_m}}\tag{10a}$$

$$E_{2s}(h_m{}^{+})=\sum_{\beta,\gamma}E^{0}{}_{vis,\beta}E^{0}{}_{ir,\gamma}\ \frac{t_{1m,vis,\beta}t_{mm'',vis,\beta}(1+r_{m''2,vis,\beta})e^{i\beta_{m,vis}h_m}}{(1+r_{mm'',vis,\beta}r_{m''2,vis,\beta})+r_{1m,vis,\beta}(r_{mm'',vis,\beta}+r_{m''2,vis,\beta})e^{2i\beta_{m,vis}h_m}}$$
$$\times\frac{t_{1m,ir,\gamma}t_{mm'',ir,\gamma}(1+r_{m''2,ir,\gamma})e^{i\beta_{m,ir}h_m}}{(1+r_{mm'',ir,\gamma}r_{m''2,ir,\gamma})+r_{1m,ir,\gamma}(r_{mm'',ir,\gamma}+r_{m''2,ir,\gamma})e^{2i\beta_{m,ir}h_m}}$$
$$\times\frac{(1+r_{mm'',SF,s}r_{m''2,SF,s})(1-r_{1m,SF,s}e^{2i\beta_{m,SF}h_m})L^{+}{}_{2/m,sy}\chi_{y\beta\gamma}}{1+r_{mm'',SF,s}r_{m''2,SF,s}+r_{1m,SF,s}(r_{mm'',SF,s}+r_{m''2,SF,s})e^{2i\beta_{m,SF}h_m}}\tag{10b}$$

m/2 界面の内側 (x = -(2n + 1)$h_m$tanθ$_{m,SF}$, z = $h_m{}^{-}$) からの反射 SFG

反射 SFG

$$E^{-}{}_{1p}(0^{-})=\sum_{\beta,\gamma}E^{0}{}_{vis,\beta}E^{0}{}_{ir,\gamma}\ \frac{t_{1m,vis,\beta}t_{1m,ir,\gamma}(1+r_{m2,vis,\beta})(1+r_{m2,ir,\gamma})e^{i(\beta_{m,ir}+\beta_{m,vis})h_m}}{(1+r_{1m,vis,\beta}r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})(1+r_{1m,ir,\gamma}r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})}$$
$$\times\frac{t_{m1,SF,p}e^{i\beta_{m,SF}h_m}}{1+r_{1m,SF,p}r_{m2,SF,p}e^{2i\beta_{m,SF}h_m}}(L^{-}{}_{2/m,px}\chi_{x\beta\gamma}+L^{-}{}_{2/m,pz}\chi_{z\beta\gamma})\tag{11a}$$

$$E^{-}{}_{1s}(0^{-})=\sum_{\beta,\gamma}E^{0}{}_{vis,\beta}E^{0}{}_{ir,\gamma}\ \frac{t_{1m,vis,\beta}t_{1m,ir,\gamma}(1+r_{m2,vis,\beta})(1+r_{m2,ir,\gamma})e^{i(\beta_{m,ir}+\beta_{m,vis})h_m}}{(1+r_{1m,vis,\beta}r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})(1+r_{1m,ir,\gamma}r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})}$$
$$\times\frac{t_{m1,SF,s}e^{i\beta_{m,SF}h_m}}{1+r_{1m,SF,s}r_{m2,SF,s}e^{2i\beta_{m,SF}h_m}}L^{-}{}_{2/m,sy}\chi_{y\beta\gamma}\tag{11b}$$

透過 SFG

$$E^{+}{}_{2p}(h_m{}^{+})=\sum_{\beta\gamma}E^{0}{}_{vis,\beta}E^{0}{}_{ir,\gamma}e\frac{t_{1m,vis,\beta}t_{1m,ir,\gamma}(1+r_{m2,vis,\beta})(1+r_{m2,ir,\gamma})e^{i(\beta_{m,ir}+\beta_{m,vis})h_m}}{(1+r_{1m,vis,\beta}r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})(1+r_{1m,ir,\gamma}r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})}$$
$$\times\frac{(1+r_{m1,SF,p}e^{2i\beta_{m,SF}h_m})L^{+}{}_{2/m,px}\chi_{x\beta\gamma}+(-1+r_{m1,SF,p}e^{2i\beta_{m,SF}h_m})L^{+}{}_{2/m,pz}\chi_{z\beta\gamma}}{1+r_{1mSF,p}r_{m2,SF,p}e^{2i\beta_{m,SF}h_m}}\tag{12a}$$

$$E^{+}_{2s}(h_m^{+}) = \sum_{\beta,\gamma} E^{0}_{vis,\beta} E^{0}_{ir,\gamma}\, e \frac{t_{1m,vis\beta}\, t_{1m,ir\gamma}\,(1+r_{m2,vis,\beta})(1+r_{m2,ir,\gamma})e^{i(\beta_{m,ir}+\beta_{m,vis})h_m}}{(1+r_{1m,vis\beta}\, r_{m2,vis,\beta} e^{2i\beta_{m,vis}h_m})(1+r_{1m,ir\gamma}\, r_{m2,ir,\gamma} e^{2i\beta_{m,ir}h_m})}$$
$$\times \frac{(1+r_{m1,SF,s} e^{2i\beta_{m,SF}h_m}) L^{+}_{2/m,sy}\chi_{y\beta\gamma}}{1+r_{1mSF,s}\, r_{m2,SF,s} e^{2i\beta_{m,SF}h_m}} \qquad (12b)$$

膜バルク [$0 < z < h_m$, $A_{n=0,1,2}: x = -2nh_m\tan\theta_{m,SF} - z\tan\theta_{m,SF}$, $B_{n=-1,0,1,2}: x = -(2n+1)h_m\tan\theta_{m,SF} + z\tan\theta_{m,SF}$] からの SFG

反射 SFG

$$E^{-}_{1p}(0^{-}) = \frac{i}{\delta}\sum_{\beta,\gamma} E^{0}_{vis,\beta} E^{0}_{ir,\gamma}$$
$$\times \frac{t_{m1,SF,p}\, t_{1m,vis,\beta}\, t_{1m,ir,\gamma}}{(1+r_{1m,SF,p}\, r_{m2,SF,p} e^{2i\beta_{m,SF}h_m})(1+r_{1m,vis,\beta}\, r_{m2,vis,\beta} e^{2i\beta_{m,ir}h_m})(1+r_{1m,ir,\gamma}\, r_{m2,ir,\gamma} e^{2i\beta_{m,vis}h_m})}$$
$$\times\{\frac{1}{\beta_{m,SF}-\beta_{m,vis}-\beta_{m,ir}}$$
$$\times\{\chi_{x\beta\gamma} L^{-}_{1/m,px}[r_{m2,vis,\beta}\, r_{m2,ir,\gamma}(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m} - e^{2i(\beta_{m,vis}+\beta_{m,ir})h_m})$$
$$+r_{m2,SF,p}(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m} - e^{2i\beta_{m,SF}h_m})]$$
$$+\chi_{z\beta\gamma} L^{-}_{1/m,pz}[r_{m2,vis,\beta}\, r_{m2,ir,\gamma}(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m} - e^{2i(\beta_{m,vis}+\beta_{m,ir})h_m})$$
$$-r_{m2,SF,p}(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m} - e^{2i\beta_{m,SF}h_m})]\}$$
$$+\frac{1}{\beta_{m,SF}-\beta_{m,vis}+\beta_{m,ir}}$$
$$\times\{\chi_{x\beta\gamma} L^{-}_{1/m,px}[r_{m2,vis,\beta}(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m} - e^{2i\beta_{m,vis}h_m})$$
$$+r_{m2,SF,p}\, r_{m2,ir,\gamma}(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m} - e^{2i(\beta_{m,SF}+\beta_{m,ir})h_m})]$$
$$+\chi_{z\beta\gamma} L^{-}_{1/m,pz}[r_{m2,vis,\beta}(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m} - e^{2i\beta_{m,vis}h_m})$$
$$-r_{m2,SF,p}\, r_{m2,ir,\gamma}(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m} - e^{2i(\beta_{m,SF}+\beta_{m,ir})h_m})]\}$$
$$+\frac{1}{\beta_{m,SF}+\beta_{m,vis}-\beta_{m,ir}}$$
$$\times\{\chi_{x\beta\gamma} L^{-}_{m/m,px}[r_{m2,ir,\gamma}(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m} - e^{2i\beta_{m,ir}h_m})$$
$$+r_{m2,SF,p}\, r_{m2,vis,\beta}(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m} - e^{2i(\beta_{m,SF}+\beta_{m,vis})h_m})]$$
$$+\chi_{z\beta\gamma} L^{-}_{m/m,pz}[r_{m2,ir,\gamma}(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m} - e^{2i\beta_{m,ir}h_m})$$
$$-r_{m2,SF,p}\, r_{m2,vis,\beta}(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m} - e^{2i(\beta_{m,SF}+\beta_{m,vis})h_m})]\}$$
$$+\frac{1}{\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir}}$$
$$\times\{\chi_{x\beta\gamma} L^{-}_{m/m,px}[(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m} - 1)$$
$$+r_{m2,SF,p}\, r_{m2,vis,\beta}\, r_{m2,ir,\gamma}(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m} - e^{2i(\beta_{m,vis}+\beta_{m,ir})h_m})]$$
$$+\chi_{z\beta\gamma} L^{-}_{m/m,pz}[(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m} - 1)$$
$$-r_{m2,SF,p}\, r_{m2,vis,\beta}\, r_{m2,ir,\gamma}(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m} - e^{2i(\beta_{m,vis}+\beta_{m,ir})h_m})]\} \qquad (13a)$$

$$E^{-}{}_{1s}(0^{-})=\frac{i}{\delta}\sum_{\beta\gamma}E^{0}{}_{vis,\beta}E^{0}{}_{ir,\gamma}$$

$$\times\frac{t_{m1,SF,s}t_{1m,vis,\beta}t_{1m,ir,\gamma}}{(1+r_{1m,SF,s}r_{m2,SF,s}e^{2i\beta_{m,SF}h_m})(1+r_{1m,vis,\beta}r_{m2,vis,\beta}e^{2i\beta_{m,ir}h_m})(1+r_{1m,ir,\gamma}r_{m2,ir,\gamma}e^{2i\beta_{m,vis}h_m})}$$

$$\times\{\frac{1}{\beta_{m,SF}-\beta_{m,vis}-\beta_{m,ir}}$$

$$\times\{\chi_{y\beta\gamma}L^{-}{}_{1/m,sy}[r_{m2,vis,\beta}r_{m2,ir,\gamma}(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}-e^{2i(\beta_{m,vis}+\beta_{m,ir})h_m})$$

$$+r_{m2,SF,s}(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}-e^{2i\beta_{m,SF}h_m})]$$

$$+\frac{1}{\beta_{m,SF}-\beta_{m,vis}+\beta_{m,ir}}$$

$$\times\{\chi_{y\beta\gamma}L^{-}{}_{1/m,sy}[r_{m2,vis,\beta}(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}-e^{2i\beta_{m,vis}h_m})$$

$$+r_{m2,SF,s}r_{m2,ir,\gamma}(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}-e^{2i(\beta_{m,SF}+\beta_{m,ir})h_m})]$$

$$+\frac{1}{\beta_{m,SF}+\beta_{m,vis}-\beta_{m,ir}}$$

$$\times\{\chi_{y\beta\gamma}L^{-}{}_{m/m,sy}[r_{m2,ir,\gamma}(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}-e^{2i\beta_{m,ir}h_m})$$

$$+r_{m2,SF,s}r_{m2,vis,\beta}(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}-e^{2i(\beta_{m,SF}+\beta_{m,vis})h_m})]$$

$$+\frac{1}{\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir}}$$

$$\times\{\chi_{y\beta\gamma}L^{-}{}_{m/m,sy}[(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}-1)$$

$$+r_{m2,SF,s}r_{m2,vis,\beta}r_{m2,ir,\gamma}(e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}-e^{2i(\beta_{m,vis}+\beta_{m,ir})h_m})] \qquad (13b)$$

透過 SFG

$$E^{+}{}_{2p}(h_m{}^{+})=\frac{i}{\delta}\sum_{\beta\gamma}E^{0}{}_{vis,\beta}E^{0}{}_{ir,\gamma}$$

$$\times\frac{t_{1m,vis,\beta}t_{1m,ir,\gamma}e^{i\beta_{m,SF}h_m}}{(1+r_{1m,SF,p}r_{m2,SF,p}e^{2i\beta_{m,SF}h_m})(1+r_{1m,vis,\beta}r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})(1+r_{1m,ir,\gamma}r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})}$$

$$\times\{L^{+}{}_{2/m,px}\chi_{x\beta\gamma}[\frac{[e^{i(-\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}-1]+r_{m1,SF,p}r_{m2,vis,\beta}r_{m2,ir,\gamma}[e^{2i(\beta_{m,vis}+\beta_{m,ir})h_m}-e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}]}{\beta_{m,SF}-\beta_{m,vis}-\beta_{m,ir}}$$

$$+\frac{r_{m2,ir,\gamma}[e^{i(-\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}-e^{2i\beta_{m,ir}h_m}]+r_{m1,SF,p}r_{m2,vis,\beta}[e^{2i\beta_{m,vis}h_m}-e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}]}{\beta_{m,SF}-\beta_{m,vis}+\beta_{m,ir}}$$

$$+\frac{r_{m2,vis,\beta}[e^{i(-\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}-e^{2i\beta_{m,vis}h_m}]+r_{m1,SF,p}r_{m2,ir,\gamma}[e^{2i\beta_{m,ir}h_m}-e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}]}{\beta_{m,SF}+\beta_{m,vis}-\beta_{m,ir}}$$

$$+\frac{r_{m2,vis,\beta}r_{m2,ir,\gamma}[e^{i(-\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}-e^{2i(\beta_{m,vis}+\beta_{m,ir})h_m}]+r_{m1,SF,p}[1-e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}]}{\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir}}]$$

$$+L^{+}{}_{2/m,pz}\chi_{z\beta\gamma}[\frac{[e^{i(-\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}-1]-r_{m1,SF,p}r_{m2,vis,\beta}r_{m2ir,\gamma}[e^{2i(\beta_{m,vis}+\beta_{m,ir})h_m}-e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}]}{\beta_{m,SF}-\beta_{m,vis}-\beta_{m,ir}}$$

$$+\frac{r_{m2,ir,\gamma}[e^{i(-\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}-e^{2i\beta_{m,ir}h_m}]-r_{m1,SF,p}r_{m2,vis,\beta}[e^{2i\beta_{m,vis}h_m}-e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}]}{\beta_{m,SF}-\beta_{m,vis}+\beta_{m,ir}}$$

$$+\frac{r_{m2,vis,\beta}[e^{i(-\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}-e^{2i\beta_{m,vis}h_m}]-r_{m1,SF,p}r_{m2,ir,\gamma}[e^{2i\beta_{m,ir}h_m}-e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}]}{\beta_{m,SF}+\beta_{m,vis}-\beta_{m,ir}}$$

$$+\frac{r_{m2,vis,\beta}r_{m2,ir,\gamma}[e^{i(-\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}-e^{2i(\beta_{m,vis}+\beta_{m,ir})h_m}]-r_{m1,SF,p}[1-e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}]}{\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir}}]\} \tag{14a}$$

$$E^{+}{}_{2s}(h_m{}^{+})=\frac{i}{\delta}\sum_{\beta,\gamma}E^{0}{}_{vis,\beta}E^{0}{}_{ir,\gamma}$$

$$\times\frac{t_{1m,vis,\beta}t_{1m,ir,\gamma}e^{i\beta_{m,SF}h_m}}{(1+r_{1m,SF,s}r_{m2,SF,s}e^{2i\beta_{m,SF}h_m})(1+r_{1m,vis,\beta}r_{m2,vis,\beta}e^{2i\beta_{m,vis}h_m})(1+r_{1m,ir,\gamma}r_{m2,ir,\gamma}e^{2i\beta_{m,ir}h_m})}$$

$$\times L^{+}{}_{2/m,sy}\chi_{y\beta\gamma}\{\frac{[e^{i(-\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}-1]+r_{m1,SF,s}r_{m2,vis,\beta}r_{m2ir,\gamma}[e^{2i(\beta_{m,vis}+\beta_{m,ir})h_m}-e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}]}{\beta_{m,SF}-\beta_{m,vis}-\beta_{m,ir}}$$

$$+\frac{r_{m2,ir,\gamma}[e^{i(-\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}-e^{2i\beta_{m,ir}h_m}]+r_{m1,SF,s}r_{m2,vis,\beta}[e^{2i\beta_{m,vis}h_m}-e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}]}{\beta_{m,SF}-\beta_{m,vis}+\beta_{m,ir}}$$

$$+\frac{r_{m2,vis,\beta}[e^{i(-\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}-e^{2i\beta_{m,vis}h_m}]+r_{m1,SF,s}r_{m2,ir,\gamma}[e^{2i\beta_{m,ir}h_m}-e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}]}{\beta_{m,SF}+\beta_{m,vis}-\beta_{m,ir}}$$

$$+\frac{r_{m2,vis,\beta}r_{m2,ir,\gamma}[e^{i(-\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}-e^{2i(\beta_{m,vis}+\beta_{m,ir})h_m}]+r_{m1,SF,s}[1-e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_m}]}{\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir}}\} \tag{14b}$$