

分子固定系から空間固定系への SFG テンソルの変換

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1. 序論

可視光が分子の電子共鳴にかかっていたり、分子がキラルであったりといった特殊なケースを除外した時の SFG テンソルについて、分子固定系の成分を空間固定系の成分に書き変える。即ち、本稿に示す SFG テンソルの XYZ 成分の表式は、分子固定 (abc) 系での成分の下付きについて左の 2 つを入れ替えたものが等しいとき ($\beta_{ijk} = \beta_{jik}$) にあてはまる一般式である。

なお、 $\beta_{ijk} = \beta_{jik}$ の関係 (ラマンテンソル部分の実数かつ対称である) は、通常非共鳴ラマン散乱が起こる系で成立する。例外は、(1) 可視光又は SFG 光が分子の電子共鳴にかかっているとき、(2) キラルな分子を扱うとき、(3) 強い外場又は近接する分子の場が有意な摂動を分子に与えているときである。

2. 分子固定座標系と空間固定座標系

1. 分子に固定した座標系 : (abc) 系と表す。
2. 空間に固定した座標系 : (XYZ) 系と表す。
3. 分子の配向 : 分子固定 (xyz) 系がオイラー角 (χ, θ, ϕ) によって空間固定 (XYZ) 系に重なるものとして、(XYZ) 系でのテンソル成分を求める。

用いられるオイラー角 (χ, θ, ϕ) については、次のように表現される。

(1) **内部回転角** ϕ : ab 面の (表面に対する) ねじれ角である。c 軸まわりの回転で ab 面を表面と垂直にするために必要な回転角、あるいは a 軸を -Z 軸の ab 面への射影に重ねるための回転角でもある。(a 軸に沿ったベクトルと X 軸に沿ったベクトルの内積がプラスになる方向で重ねる。) ab 面が表面に垂直なときには $\phi = 0$ or π であり、ab 面が表面と向き合っているときには $\phi = \pi/2$ or $3\pi/2$ である。対応するオイラー角も同じく ϕ である。分子がランダムな内部回転角を取っている場合には ϕ は $0 \sim 2\pi$ の任意の値を同じウェイトで取る。

(2) **傾き角・tilt 角** θ_{tilt} : 通常定義に合わせて、c 軸と外向きの法線(-Z)の間の角とし、N 軸まわりの回転で c 軸を外向きの法線に重ねる方向をプラス回転とする。Z 軸は下向きの法線であるから、対応するオイラー角 θ は $\pi - \theta_{\text{tilt}}$ である。

(3) **面内配向角** $\chi_{\text{in-plane}}$ (χ_{ip} と略記) : Z 軸まわりの回転で c 軸の(XY)面への射影を X 軸に重ねるための回転角とする。ここでの Z 軸の向きでは X 軸の方向に見て射影が左側にあるときにプラスになる。Z 軸を基板の内部に向けて取っているため、対応するオイラー角 χ は $\pi/2 + \chi_{\text{ip}}$ である。分子の面内配向がランダムなときには、 χ_{ip} は $0 \sim 2\pi$ の任意の値を同じウェイトで取る。

3. 振動バンド

SFG 活性な振動は a、b、c 軸に沿った成分を持つ。そこで、その成分ごとに分類して示す。但し、分子の形や分子軸のえらびかたによっては 2 個以上の軸成分にまたがる場合があることを注意しておく。

4. 空間固定(XYZ)系におけるテンソル成分

分子固定 (xyz) 系がオイラー角 (χ, θ, ϕ) によって空間固定(XYZ)系に重なるものとして、(XYZ) 系でのテンソル成分を求める。

なお、分子固定系での成分に下の関係を仮定する：(ノンゼロ成分と相互の間系)

$$\begin{aligned} \beta_{aac}, \beta_{bbc}, \beta_{ccc}, \beta_{abc} &= \beta_{bac}, \beta_{acc} = \beta_{cac}, \beta_{bcc} = \beta_{cbc} \\ \beta_{aaa}, \beta_{bba}, \beta_{cca}, \beta_{baa} &= \beta_{aba}, \beta_{caa} = \beta_{aca}, \beta_{bca} = \beta_{cba} \\ \beta_{bbb}, \beta_{aab}, \beta_{cbb}, \beta_{cbb} &= \beta_{bcb}, \beta_{cab} = \beta_{acb}, \beta_{abb} = \beta_{bab} \end{aligned}$$

[c 軸に沿った振動]

$$\begin{aligned} (\text{ppp}) \quad \chi_{XXX} &= -(1/2)(\beta_{aac} + \beta_{bbc})\sin\theta\cos\chi \\ &+ (1/8)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})\sin^3\theta(3\cos\chi + \cos3\chi) \\ &+ (1/8)(\beta_{aac} - \beta_{bbc})\{\sin\theta(\cos\chi - \cos3\chi) - (\sin\theta - \sin^3\theta)(3\cos\chi + \cos3\chi)\}\cos2\phi \\ &+ 2\sin\theta\cos\theta(\sin\chi + \sin3\chi)\sin2\phi \\ &+ (1/4)\beta_{abc}\{\sin\theta(\cos\chi - \cos3\chi) - (\sin\theta - \sin^3\theta)(3\cos\chi + \cos3\chi)\}\sin2\phi \\ &- 2\sin\theta\cos\theta(\sin\chi + \sin3\chi)\cos2\phi \\ &+ (1/2)\beta_{aac}[(\cos\theta - \cos^3\theta)(3\cos\chi + \cos3\chi)\cos\phi - \sin^2\theta(\sin\chi + \sin3\chi)\sin\phi] \\ &+ (1/2)\beta_{bcc}[(\cos\theta - \cos^3\theta)(3\cos\chi + \cos3\chi)\sin\phi + \sin^2\theta(\sin\chi + \sin3\chi)\cos\phi] \\ \chi_{XZZ} &= (1/2)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})(\sin\theta - \sin^3\theta)\cos\chi \\ &+ (1/2)(\beta_{aac} - \beta_{bbc})[(\sin\theta - \sin^3\theta)\cos\chi\cos2\phi - \sin\theta\cos\theta\sin\chi\sin2\phi] \\ &+ \beta_{abc}[(\sin\theta - \sin^3\theta)\cos\chi\sin2\phi + \sin\theta\cos\theta\sin\chi\cos2\phi] \\ &- \beta_{aac}[(\cos\theta - 2\cos^3\theta)\cos\chi\cos\phi - \sin^2\theta\sin\chi\sin\phi] \\ &- \beta_{bcc}[(\cos\theta - 2\cos^3\theta)\cos\chi\sin\phi - \cos^2\theta\sin\chi\cos\phi] \\ \chi_{ZZX} &= (1/2)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})(\sin\theta - \sin^3\theta)\cos\chi \\ &+ (1/2)(\beta_{aac} - \beta_{bbc})[(\sin\theta - \sin^3\theta)\cos\chi\cos2\phi - \sin\theta\cos\theta\sin\chi\sin2\phi] \\ &+ \beta_{abc}[(\sin\theta - \sin^3\theta)\cos\chi\sin2\phi + \sin\theta\cos\theta\sin\chi\cos2\phi] \\ &- \beta_{aac}[(\cos\theta - 2\cos^3\theta)\cos\chi\cos\phi - \sin^2\theta\sin\chi\sin\phi] \\ &- \beta_{bcc}[(\cos\theta - 2\cos^3\theta)\cos\chi\sin\phi - \cos^2\theta\sin\chi\cos\phi] \\ \chi_{ZZX} &= -(1/2)(\beta_{aac} + \beta_{bbc})\sin\theta\cos\chi \\ &+ (1/2)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})(\sin\theta - \sin^3\theta)\cos\chi \\ &- (1/2)(\beta_{aac} - \beta_{bbc})\cos\chi\sin^3\theta\cos2\phi \\ &- \beta_{abc}(2\sin\theta - \sin^3\theta)\cos\chi\sin2\phi \\ &- 2\beta_{aac}(\cos\theta - 2\cos^3\theta)\cos\chi\cos\phi \\ &- 2\beta_{bcc}(\cos\theta - 2\cos^3\theta)\cos\chi\sin\phi \\ \chi_{XZX} &= -(1/4)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})(\cos\theta - \cos^3\theta)(1 + \cos2\chi) \\ &- (1/4)(\beta_{aac} - \beta_{bbc})[(\cos\theta - \cos^3\theta)(1 + \cos2\chi)\cos2\phi - \sin^2\theta\sin2\chi\sin2\phi] \\ &- (1/2)\beta_{abc}[(\cos\theta - \cos^3\theta)(1 + \cos2\chi)\sin2\phi + \sin^2\theta\sin2\chi\cos2\phi] \\ &- (1/2)\beta_{aac}[(\sin\theta - 2\sin^3\theta)(1 + \cos2\chi)\cos\phi - \sin\theta\cos\theta\sin2\chi\sin\phi] \\ &- (1/2)\beta_{bcc}[(\sin\theta - 2\sin^3\theta)(1 + \cos2\chi)\sin\phi + \sin\theta\cos\theta\sin2\chi\cos\phi] \\ \chi_{ZXX} &= -(1/4)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})(\cos\theta - \cos^3\theta)(1 + \cos2\chi) \\ &- (1/4)(\beta_{aac} - \beta_{bbc})[(\cos\theta - \cos^3\theta)(1 + \cos2\chi)\cos2\phi - \sin^2\theta\sin2\chi\sin2\phi] \\ &- (1/2)\beta_{abc}[(\cos\theta - \cos^3\theta)(1 + \cos2\chi)\sin2\phi + \sin^2\theta\sin2\chi\cos2\phi] \end{aligned}$$

$$\begin{aligned}
& - (1/2)\beta_{acc}[(\sin\theta - 2\sin^3\theta)(1 + \cos 2\chi)\cos\phi - \sin\theta\cos\theta\sin 2\chi\sin\phi] \\
& - (1/2)\beta_{bcc}[(\sin\theta - 2\sin^3\theta)(1 + \cos 2\chi)\sin\phi + \sin\theta\cos\theta\sin 2\chi\cos\phi] \\
\chi_{XXZ} = & (1/2)(\beta_{aac} + \beta_{bbc})\cos\theta \\
& - (1/4)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})(\cos\theta - \cos^3\theta)(1 + \cos 2\chi) \\
& - (1/4)(\beta_{aac} - \beta_{bbc})\{[(\cos\theta - \cos^3\theta) - (\cos\theta + \cos^3\theta)\cos 2\chi]\cos 2\phi + 2\cos^2\theta\sin 2\chi\sin 2\phi\} \\
& - (1/2)\beta_{abc}\{[(\cos\theta - \cos^3\theta) - (\cos\theta + \cos^3\theta)\cos 2\chi]\sin 2\phi - 2\cos^2\theta\sin 2\chi\cos 2\phi\} \\
& - \beta_{aac}[(\sin\theta - \sin^3\theta)(1 + \cos 2\chi)\cos\phi - \sin\theta\cos\theta\sin 2\chi\sin\phi] \\
& - \beta_{bcc}[(\sin\theta - \sin^3\theta)(1 + \cos 2\chi)\sin\phi + \sin\theta\cos\theta\sin 2\chi\cos\phi] \\
\chi_{ZZZ} = & (1/2)(\beta_{aac} + \beta_{bbc})\cos\theta \\
& - (1/2)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})\cos^3\theta \\
& + (1/2)(\beta_{aac} - \beta_{bbc})(\cos\theta - \cos^3\theta)\cos 2\phi \\
& + \beta_{abc}(\cos\theta - \cos^3\theta)\sin 2\phi \\
& + 2\beta_{aac}(\sin\theta - \sin^3\theta)\cos\phi \\
& + 2\beta_{bcc}(\sin\theta - \sin^3\theta)\sin\phi
\end{aligned}$$

$$\begin{aligned}
(\text{spp}) \quad \chi_{YXX} = & -(1/8)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})\sin^3\theta(\sin\chi + \sin 3\chi) \\
& + (1/8)(\beta_{aac} - \beta_{bbc})[(2\sin\theta - \sin^3\theta)(\sin\chi + \sin 3\chi)\cos 2\phi + 2\sin\theta\cos\theta(\cos\chi + \cos 3\chi)\sin 2\phi] \\
& + (1/2)\beta_{abc}[2(2\sin\theta - \sin^3\theta)(\sin\chi + \sin 3\chi)\sin 2\phi - \sin\theta\cos\theta(\cos\chi + \cos 3\chi)\cos 2\phi] \\
& - (1/2)\beta_{aac}[(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)\cos\phi + \sin^2\theta(\cos\chi + \cos 3\chi)\sin\phi] \\
& - (1/2)\beta_{bcc}[(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)\sin\phi - \sin^2\theta(\cos\chi + \cos 3\chi)\cos\phi]
\end{aligned}$$

$$\begin{aligned}
\chi_{YYZ} = & -(1/2)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})(\sin\theta - \sin^3\theta)\sin\chi \\
& - (1/2)(\beta_{aac} - \beta_{bbc})[(\sin\theta - \sin^3\theta)\sin\chi\cos 2\phi + \sin\theta\cos\theta\cos\chi\sin 2\phi] \\
& - \beta_{abc}[(\sin\theta - \sin^3\theta)\sin\chi\sin 2\phi - \sin\theta\cos\theta\cos\chi\cos 2\phi] \\
& + \beta_{aac}[(\cos\theta - 2\cos^3\theta)\sin\chi\cos\phi - \cos^2\theta\cos\chi\sin\phi] \\
& + \beta_{bcc}[(\cos\theta - 2\cos^3\theta)\sin\chi\sin\phi + \cos^2\theta\cos\chi\cos\phi]
\end{aligned}$$

$$\begin{aligned}
\chi_{YXZ} = & (1/4)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})(\cos\theta - \cos^3\theta)\sin 2\chi \\
& + (1/4)(\beta_{aac} - \beta_{bbc})[(\cos\theta - \cos^3\theta)\sin 2\chi\cos 2\phi + \sin^2\theta(1 + \cos 2\chi)\sin 2\phi] \\
& + (1/2)\beta_{abc}[(\cos\theta - \cos^3\theta)\sin 2\chi\sin 2\phi - \sin^2\theta(1 + \cos 2\chi)\cos 2\phi] \\
& + (1/2)\beta_{aac}[(\sin\theta - 2\sin^3\theta)\cos 2\chi\cos\phi + \sin 2\theta(1 + \cos 2\chi)\sin\phi] \\
& + (1/2)\beta_{bcc}[(\sin\theta - 2\sin^3\theta)\cos 2\chi\sin\phi - \sin 2\theta(1 + \cos 2\chi)\cos\phi]
\end{aligned}$$

$$\begin{aligned}
\chi_{XXZ} = & (1/4)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})(\cos\theta - \cos^3\theta)\sin 2\chi \\
& - (1/4)(\beta_{aac} - \beta_{bbc})[(\cos\theta + \cos^3\theta)\sin 2\chi\cos 2\phi + 2\cos^2\theta\cos 2\chi\sin 2\phi] \\
& - (1/2)\beta_{abc}[(\cos\theta + \cos^3\theta)\sin 2\chi\sin 2\phi - 2\cos^2\theta\cos 2\chi\cos 2\phi] \\
& + \beta_{aac}[(\sin\theta - \sin^3\theta)\sin 2\chi\cos\phi + \sin\theta\cos\theta\cos 2\chi\sin\phi] \\
& + \beta_{bcc}[(\sin\theta - \sin^3\theta)\sin 2\chi\sin\phi - \sin\theta\cos\theta\cos 2\chi\cos\phi]
\end{aligned}$$

$$\begin{aligned}
(\text{ssp}) \quad \chi_{YYX} = & -(1/2)(\beta_{aac} + \beta_{bbc})\sin\theta\cos\chi \\
& + (1/8)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})\sin^3\theta(\cos\chi - \cos 3\chi) \\
& + (1/8)(\beta_{aac} - \beta_{bbc})\{[\sin\theta(3\cos\chi + \cos 3\chi) - (\sin\theta - \sin^3\theta)(\cos\chi - \cos 3\chi)]\cos 2\phi \\
& \quad - 2\sin\theta\cos\theta(\sin\chi + \sin 3\chi)\sin 2\phi\} \\
& + (1/4)\beta_{abc}\{[4\sin\theta\cos\chi - (2\sin\theta - \sin^3\theta)]\sin 2\phi + 2\sin\theta\cos\theta(\sin\chi + \sin 3\chi)\cos 2\phi\}
\end{aligned}$$

$$\begin{aligned}
& + (1/2)\beta_{acc}[(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)\cos\phi + \sin^2\theta(\sin\chi + \sin 3\chi)\sin\phi] \\
& + (1/2)\beta_{bcc}[(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)\sin\phi - \sin^2\theta(\sin\chi + \sin 3\chi)\cos\phi] \\
\chi_{YYZ} = & (1/2)(\beta_{aac} + \beta_{bbc})\cos\theta \\
& - (1/4)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})(\cos\theta - \cos^3\theta)(1 - \cos 2\chi) \\
& - (1/4)(\beta_{aac} - \beta_{bbc})\{[(\cos\theta - \cos^3\theta) + (\cos\theta + \cos^3\theta)\cos 2\chi]\cos 2\phi - 2\cos^2\theta\sin 2\chi\sin 2\phi\} \\
& - (1/2)\beta_{abc}\{[(\cos\theta - \cos^3\theta) + (\cos\theta + \cos^3\theta)\cos 2\chi]\sin 2\phi + \cos^2\theta\sin 2\chi\cos 2\phi\} \\
& - \beta_{acc}[(\sin\theta - \sin^3\theta)(1 - \cos 2\chi)\cos\phi + \sin\theta\cos\theta\sin 2\chi\sin\phi] \\
& - \beta_{bcc}[(\sin\theta - \sin^3\theta)(1 - \cos 2\chi)\sin\phi - \sin\theta\cos\theta\sin 2\chi\cos\phi] \\
(\text{psp}) \quad \chi_{XXY} = & -(1/8)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})\sin^3\theta(\sin\chi + \sin 3\chi) \\
& + (1/8)(\beta_{aac} - \beta_{bbc})[(2\sin\theta - \sin^3\theta)(\sin\chi + \sin 3\chi)\cos 2\phi + 2\sin\theta\cos\theta(\cos\chi + \cos 3\chi)\sin 2\phi] \\
& + (1/4)\beta_{abc}[(2\sin\theta - \sin^3\theta)(\sin\chi + \sin 3\chi)\sin 2\phi - 2\sin\theta\cos\theta(\cos\chi + \cos 3\chi)\cos 2\phi] \\
& - (1/2)\beta_{acc}[(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)\cos\phi + \sin^2\theta(\cos\chi + \cos 3\chi)\sin\phi] \\
& - (1/2)\beta_{bcc}[(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)\sin\phi - \sin^2\theta(\cos\chi + \cos 3\chi)\cos\phi] \\
\chi_{ZZY} = & -(1/2)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})(\sin\theta - \sin^3\theta)\sin\chi \\
& - (1/2)(\beta_{aac} - \beta_{bbc})[(\sin\theta - \sin^3\theta)\sin\chi \cos 2\phi + \sin\theta\cos\theta\cos\chi\sin 2\phi] \\
& - \beta_{abc}[(\sin\theta - \sin^3\theta)\sin\chi\sin 2\phi - \sin\theta\cos\theta\cos\chi\cos 2\phi] \\
& + \beta_{acc}[(\cos\theta - 2\cos^3\theta)\sin\chi\cos\phi - \cos^2\theta\cos\chi\sin\phi] \\
& + \beta_{bcc}[(\cos\theta - 2\cos^3\theta)\sin\chi\sin\phi + \cos^2\theta\cos\chi\cos\phi] \\
\chi_{XYZ} = & (1/4)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})(\cos\theta - \cos^3\theta)\sin 2\chi \\
& - (1/4)(\beta_{aac} - \beta_{bbc})[(\cos\theta + \cos^3\theta)\sin 2\chi \cos 2\phi + 2\cos^2\theta\cos 2\chi\sin 2\phi] \\
& - (1/2)\beta_{abc}[(\cos\theta + \cos^3\theta)\sin 2\chi\sin 2\phi - 2\cos^2\theta\cos 2\chi\cos 2\phi] \\
& + \beta_{acc}[(\sin\theta - \sin^3\theta)\sin 2\chi\cos\phi + \sin\theta\cos\theta\cos 2\chi\sin\phi] \\
& + \beta_{bcc}[(\sin\theta - \sin^3\theta)\sin 2\chi\sin\phi - \sin\theta\cos\theta\cos 2\chi\cos\phi] \\
\chi_{ZXY} = & (1/4)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})(\cos\theta - \cos^3\theta)\sin 2\chi \\
& + (1/4)(\beta_{aac} - \beta_{bbc})[(\cos\theta - \cos^3\theta)\sin 2\chi \cos 2\phi + \sin^2\theta(1 + \cos 2\chi)\sin 2\phi] \\
& + (1/2)\beta_{abc}[(\cos\theta - \cos^3\theta)\sin 2\chi\sin 2\phi - \sin^2\theta(1 + \cos 2\chi)\cos 2\phi] \\
& + \beta_{acc}[(\sin\theta - \sin^3\theta)\sin 2\chi\cos\phi + \sin\theta\cos\theta\cos 2\chi\sin\phi] \\
& + \beta_{bcc}[(\sin\theta - \sin^3\theta)\sin 2\chi\sin\phi - \sin\theta\cos\theta\cos 2\chi\cos\phi] \\
(\text{sp}) \quad \chi_{XXY} = & (1/8)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})\sin^3\theta(\cos\chi - \cos 3\chi) \\
& - (1/8)(\beta_{aac} - \beta_{bbc})[(2\sin\theta - \sin^3\theta)(\cos\chi - \cos 3\chi)\cos 2\phi - 2\sin\theta\cos\theta(\sin\chi - \sin 3\chi)\sin 2\phi] \\
& - (1/4)\beta_{abc}[(2\sin\theta - \sin^3\theta)(\cos\chi - \cos 3\chi)\sin 2\phi + 2\sin\theta\cos\theta(\sin\chi + \sin 3\chi)\cos 2\phi] \\
& + (1/2)\beta_{acc}[(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)\cos\phi - \sin^2\theta(\sin\chi - \sin 3\chi)\sin\phi] \\
& + (1/2)\beta_{bcc}[(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)\sin\phi + \sin^2\theta(\sin\chi - \sin 3\chi)\cos\phi] \\
\chi_{YYZ} = & -(1/4)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})(\cos\theta - \cos^3\theta)(1 - \cos 2\chi) \\
& - (1/4)(\beta_{aac} - \beta_{bbc})[(\cos\theta - \cos^3\theta)(1 - \cos 2\chi)\cos 2\phi + \sin^2\theta\sin 2\chi\sin 2\phi] \\
& - (1/2)\beta_{abc}[(\cos\theta - \cos^3\theta)(1 - \cos 2\chi)\sin 2\phi - \sin^2\theta\sin 2\chi\cos 2\phi] \\
& - (1/2)\beta_{acc}[(\sin\theta - 2\sin^3\theta)(1 - \cos 2\chi)\cos\phi + \sin\theta\cos\theta\sin 2\chi\sin 2\phi] \\
& - (1/2)\beta_{bcc}[(\sin\theta - 2\sin^3\theta)(1 - \cos 2\chi)\sin\phi - \sin\theta\cos\theta\sin 2\chi\cos 2\phi] \\
(\text{pps}) \quad \chi_{XXY} = & (1/2)(\beta_{aac} + \beta_{bbc})\sin\theta\sin\chi
\end{aligned}$$

$$\begin{aligned}
& - (1/8)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})\sin^3\theta(\sin\chi + \sin 3\chi) \\
& + (1/8)(\beta_{aac} - \beta_{bbc})\{[(\sin\theta - \sin^3\theta)(\sin\chi + \sin 3\chi) - \sin\theta(3\sin\chi - \sin 3\chi)\cos 2\phi] \\
& \quad - 2\sin\theta\cos\theta(\cos\chi - \cos 3\chi)\sin 2\phi\} \\
& + (1/4)\beta_{abc}\{[(2\sin\theta - \sin^3\theta)(\sin\chi + \sin 3\chi) - 4\sin\theta\cos\chi]\sin 2\phi \\
& \quad + 2\sin\theta\cos\theta(\cos\chi - \cos 3\chi)\cos 2\phi\} \\
& - (1/2)\beta_{acc}[(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)\cos\phi - \sin^2\theta(\cos\chi - \cos 3\chi)\sin\phi] \\
& - (1/2)\beta_{bcc}[(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)\sin\phi + \sin^2\theta(\cos\chi - \cos 3\chi)\cos\phi] \\
\chi_{ZZY} = & (1/2)(\beta_{aac} + \beta_{bbc})\sin\theta\sin\chi \\
& - (1/2)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})(\sin\theta - \sin^3\theta)\sin\chi \\
& + (1/2)(\beta_{aac} - \beta_{bbc})\sin^3\theta\sin\chi\cos 2\phi \\
& + \beta_{abc}\sin^3\theta\sin\chi\sin 2\phi \\
& + 2\beta_{acc}(\cos\theta - \cos^3\theta)\sin\chi\cos\phi \\
& + 2\beta_{bcc}(\cos\theta - \cos^3\theta)\sin\chi\sin\phi \\
\chi_{XZY} = & (1/4)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})(\cos\theta - \cos^3\theta)\sin 2\chi \\
& + (1/4)(\beta_{aac} - \beta_{bbc})[(\cos\theta - \cos^3\theta)\sin 2\chi\cos 2\phi - \sin^2\theta(1 - \cos 2\chi)\sin 2\phi] \\
& + (1/2)\beta_{abc}[(\cos\theta - \cos^3\theta)\sin 2\chi\sin 2\phi + \sin^2\theta(1 - \cos 2\chi)\cos 2\phi] \\
& + (1/2)\beta_{acc}[(\sin\theta - 2\sin^3\theta)\sin 2\chi\cos\phi - \sin\theta\cos\theta(1 - \cos 2\chi)\sin\phi] \\
& + (1/2)\beta_{bcc}[(\sin\theta - 2\sin^3\theta)\sin 2\chi\sin\phi + \sin\theta\cos\theta(1 - \cos 2\chi)\cos\phi] \\
\chi_{ZXY} = & (1/4)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})(\cos\theta - \cos^3\theta)\sin 2\chi \\
& + (1/4)(\beta_{aac} - \beta_{bbc})[(\cos\theta - \cos^3\theta)\sin 2\chi\cos 2\phi - \sin^2\theta(1 - \cos 2\chi)\sin 2\phi] \\
& + (1/2)\beta_{abc}[(\cos\theta - \cos^3\theta)\sin 2\chi\sin 2\phi + \sin^2\theta(1 - \cos 2\chi)\cos 2\phi] \\
& + (1/2)\beta_{acc}[(\sin\theta - 2\sin^3\theta)\sin 2\chi\cos\phi - \sin\theta\cos\theta(1 - \cos 2\chi)\sin\phi] \\
& + (1/2)\beta_{bcc}[(\sin\theta - 2\sin^3\theta)\sin 2\chi\sin\phi + \sin\theta\cos\theta(1 - \cos 2\chi)\cos\phi]
\end{aligned}$$

(pss)

$$\begin{aligned}
\chi_{XYX} = & (1/8)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})\sin^3\theta(\cos\chi - \cos 3\chi) \\
& - (1/8)(\beta_{aac} - \beta_{bbc})[(2\sin\theta - \sin^3\theta)(\cos\chi - \cos 3\chi)\cos 2\phi - 2\sin\theta\cos\theta(\sin\chi - \sin 3\chi)\sin 2\phi] \\
& - (1/4)\beta_{abc}[(2\sin\theta - \sin^3\theta)(\cos\chi - \cos 3\chi)\sin 2\phi + 2\sin\theta\cos\theta(\sin\chi - \sin 3\chi)\cos 2\phi] \\
& + (1/2)\beta_{acc}[(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)\cos\phi - \sin^2\theta(\sin\chi - \sin 3\chi)\sin\phi] \\
& + (1/2)\beta_{bcc}[(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)\sin\phi + \sin^2\theta(\sin\chi - \sin 3\chi)\cos\phi] \\
\chi_{ZYY} = & -(1/4)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})(\cos\theta - \cos^3\theta)(1 - \cos 2\chi) \\
& - (1/4)(\beta_{aac} - \beta_{bbc})[(\cos\theta - \cos^3\theta)(1 - \cos 2\chi)\cos 2\phi + \sin^2\theta\sin 2\chi\sin 2\phi] \\
& - (1/2)\beta_{abc}[(\cos\theta - \cos^3\theta)(1 - \cos 2\chi)\sin 2\phi - \sin^2\theta\sin 2\chi\cos 2\phi] \\
& - (1/2)\beta_{acc}[(\sin\theta - 2\sin^3\theta)(1 - \cos 2\chi)\cos\phi + \sin\theta\cos\theta\sin 2\chi\sin\phi] \\
& - (1/2)\beta_{bcc}[(\sin\theta - 2\sin^3\theta)(1 - \cos 2\chi)\sin\phi - \sin\theta\cos\theta\sin 2\chi\cos\phi]
\end{aligned}$$

(sss)

$$\begin{aligned}
\chi_{YYX} = & (1/2)(\beta_{aac} + \beta_{bbc})\sin\theta\sin\chi \\
& - (1/8)(\beta_{aac} + \beta_{bbc} - 2\beta_{ccc})\sin^3\theta(3\sin\chi - \sin 3\chi) \\
& - (1/8)(\beta_{aac} - \beta_{bbc})\{[\sin\theta(\sin\chi + \sin 3\chi) - (\sin\theta - \sin^3\theta)(3\sin\chi - \sin 3\chi)]\cos 2\phi \\
& \quad - 2\sin\theta\cos\theta(\cos\chi - \cos 3\chi)\sin 2\phi\} \\
& + (1/4)\beta_{abc}\{[4(\sin\theta - \sin^3\theta)\sin\chi - (2\sin\theta - \sin^3\theta)(\sin\chi + \sin 3\chi)]\sin 2\phi \\
& \quad - 2\sin\theta\cos\theta(\cos\chi - \cos 3\chi)\cos 2\phi\}
\end{aligned}$$

$$\begin{aligned}
& - (1/2)\beta_{acc}[(\cos\theta - \cos^3\theta)(3\sin\chi - \sin 3\chi)\cos\phi + \sin^2\theta(\cos\chi - \cos 3\chi)\sin\phi] \\
& - (1/2)\beta_{bcc}[(\cos\theta - \cos^3\theta)(3\sin\chi - \sin 3\chi)\sin\phi - \sin^2\theta(\cos\chi - \cos 3\chi)\cos\phi]
\end{aligned}$$

[a 軸に沿った振動]

$$\begin{aligned}
(\text{ppp}) \quad \chi_{xxx} = & -(1/4)\beta_{caa}\{[(\sin\theta - \sin^3\theta)(3\cos\chi + \cos 3\chi)(1 + \cos 2\phi) + \sin\theta(\cos\chi - \cos 3\chi)(1 - \cos 2\phi)] \\
& - 2\sin\theta\cos\theta(\sin\chi + \sin 3\chi)\sin 2\phi\} \\
& + (1/4)\beta_{bca}\{[\sin\theta(\cos\chi - \cos 3\chi) - (\sin\theta - \sin^3\theta)(3\cos\chi + \cos 3\chi)]\sin 2\phi \\
& - 2\sin\theta\cos\theta(\sin\chi + \sin 3\chi)\cos 2\phi\} \\
& + (1/16)\beta_{aaa}[-(\cos\theta - \cos^3\theta)(3\cos\chi + \cos 3\chi)(3\cos\phi + \cos 3\phi) \\
& + 4\cos\theta(3\cos\chi\cos\phi + \cos 3\chi\cos 3\phi) - 4(3\sin\chi\sin\phi + \sin 3\chi\sin 3\phi) \\
& + 3\sin^2\theta(\sin\chi + \sin 3\chi)(\sin\phi + \sin 3\phi)] \\
& + (1/16)\beta_{bba}[-(\cos\theta - \cos^3\theta)(3\cos\chi + \cos 3\chi)(\cos\phi - \cos 3\phi) \\
& + 4\cos\theta(\cos\chi\cos\phi - \cos 3\chi\cos 3\phi) - 4(\sin\chi\sin\phi - \sin 3\chi\sin 3\phi) \\
& + \sin^2\theta(\sin\chi + \sin 3\chi)(\sin\phi - \sin 3\phi)] \\
& + (1/4)\beta_{cca}[(\cos\theta - \cos^3\theta)(3\cos\chi + \cos 3\chi)\cos\phi - \sin^2\theta(\sin\chi + \sin 3\chi)\sin\phi] \\
\chi_{xzz} = & (1/2)\beta_{caa}[(\sin\theta - 2\sin^3\theta)\cos\chi(1 + \cos 2\phi) - \sin\theta\cos\theta\sin\chi\sin 2\phi] \\
& + (1/2)\beta_{bca}[(\sin\theta - 2\sin^3\theta)\cos\chi\sin 2\phi + \sin\theta\cos\theta\sin\chi(1 + \cos 2\phi)] \\
& + (1/4)\beta_{aaa}[(\cos\theta - \cos^3\theta)\cos\chi(3\cos\phi + \cos 3\phi) - \sin^2\theta\sin\chi(\sin\phi + \sin 3\phi)] \\
& + (1/4)\beta_{bba}[(\cos\theta - \cos^3\theta)\cos\chi(\cos\phi - \cos 3\phi) + \sin^2\theta\sin\chi(\sin\phi + \sin 3\phi)] \\
& - \beta_{cca}(\cos\theta - \cos^3\theta)\cos\chi\cos\phi \\
\chi_{zxx} = & (1/2)\beta_{caa}[(\sin\theta - 2\sin^3\theta)\cos\chi(1 + \cos 2\phi) - \sin\theta\cos\theta\sin\chi\sin 2\phi] \\
& + (1/2)\beta_{bca}[(\sin\theta - 2\sin^3\theta)\cos\chi\sin 2\phi + \sin\theta\cos\theta\sin\chi(1 + \cos 2\phi)] \\
& + (1/4)\beta_{aaa}[(\cos\theta - \cos^3\theta)\cos\chi(3\cos\phi + \cos 3\phi) - \sin^2\theta\sin\chi(\sin\phi + \sin 3\phi)] \\
& + (1/4)\beta_{bba}[(\cos\theta - \cos^3\theta)\cos\chi(\cos\phi - \cos 3\phi) + \sin^2\theta\sin\chi(\sin\phi + \sin 3\phi)] \\
& - \beta_{cca}(\cos\theta - \cos^3\theta)\cos\chi\cos\phi \\
\chi_{zzx} = & \beta_{caa}[(\sin\theta - \sin^3\theta)\cos\chi(1 + \cos 2\phi) - \sin\theta\cos\theta\sin\chi\sin 2\phi] \\
& + \beta_{bca}[(\sin\theta - \sin^3\theta)\cos\chi\sin 2\phi - \sin\theta\cos\theta\sin\chi(1 - \cos 2\phi)] \\
& + (1/4)\beta_{aaa}[(\cos\theta - \cos^3\theta)\cos\chi(3\cos\phi + \cos 3\phi) - \sin^2\theta\sin\chi(\sin\phi + \sin 3\phi)] \\
& + (1/4)\beta_{bba}[(\cos\theta - \cos^3\theta)\cos\chi(\cos\phi - \cos 3\phi) - \sin^2\theta\sin\chi(3\sin\phi - \sin 3\phi)] \\
& + \beta_{cca}[\cos^3\theta\cos\chi\cos\phi - \cos^2\theta\sin\chi\sin\phi] \\
\chi_{zxx} = & (1/4)\beta_{caa}\{2[\cos\theta(1 + \cos 2\phi\cos 2\chi) - (\cos\theta - \cos^3\theta)(1 + \cos 2\chi)(1 + \cos 2\phi)] \\
& + (1 - 3\cos^2\theta)\sin 2\chi\sin 2\phi\} \\
& - (1/4)\beta_{bca}\{2[(\cos\theta - \cos^3\theta) - \cos^3\theta\cos 2\chi]\sin 2\phi - [\sin^2\theta - (1 - 3\cos^2\theta)\cos 2\phi]\sin 2\chi\} \\
& + (1/8)\beta_{aaa}[(\sin\theta - \sin^3\theta)(1 + \cos 2\chi)(3\cos\phi + \cos 3\phi) + \sin\theta(1 - \cos 2\chi)(\cos\phi - \cos 3\phi) \\
& - 2\sin\theta\cos\theta\sin 2\chi(\sin\phi + \sin 3\phi)] \\
& + (1/8)\beta_{bba}[(\sin\theta - \sin^3\theta)(1 + \cos 2\chi)(\cos\phi - \cos 3\phi) - \sin\theta(1 - \cos 2\chi)(\cos\phi - \cos 3\phi) \\
& - 2\sin\theta\cos\theta\sin 2\chi(\sin\phi - \sin 3\phi)] \\
& + (1/2)\beta_{cca}[-(\sin\theta - \sin^3\theta)(1 + \cos 2\chi)\cos\phi + \sin\theta\cos\theta\sin 2\chi\sin\phi] \\
\chi_{xzx} = & (1/4)\beta_{caa}\{2[\cos\theta(1 + \cos 2\phi\cos 2\chi) - (\cos\theta - \cos^3\theta)(1 + \cos 2\chi)(1 + \cos 2\phi)] \\
& + (1 - 3\cos^2\theta)\sin 2\chi\sin 2\phi\} \\
& - (1/4)\beta_{bca}\{2[(\cos\theta - \cos^3\theta) - \cos^3\theta\cos 2\chi]\sin 2\phi - [\sin^2\theta - (1 - 3\cos^2\theta)\cos 2\phi]\sin 2\chi\}
\end{aligned}$$

$$\begin{aligned}
& + (1/8)\beta_{aaa}[\sin\theta(1 - \cos 2\chi)(\cos\phi - \cos 3\phi) + (\sin\theta - \sin^3\theta)(1 + \cos 2\chi)(3\cos\phi + \cos 3\phi) \\
& \quad - 2\sin\theta\cos\theta\sin 2\chi(\sin\phi + \sin 3\phi)] \\
& + (1/8)\beta_{bba}[-\sin\theta(1 - \cos 2\chi)(\cos\phi - \cos 3\phi) + (\sin\theta - \sin^3\theta)(1 + \cos 2\chi)(\cos\phi - \cos 3\phi) \\
& \quad - 2\sin\theta\cos\theta\sin 2\chi(\sin\phi - \sin 3\phi)] \\
& + (1/2)\beta_{cca}[-(\sin\theta - \sin^3\theta)(1 + \cos 2\chi)\cos\phi + \sin\theta\cos\theta\sin 2\chi\sin\phi] \\
\chi_{xxx} = & -(1/2)\beta_{caa}[(\cos\theta - \cos^3\theta)(1 + \cos 2\chi)(1 + \cos 2\phi) - \sin^2\theta\sin 2\chi\sin 2\phi] \\
& - (1/2)\beta_{bca}[(\cos\theta - \cos^3\theta)(1 + \cos 2\chi)\sin 2\phi + \sin^2\theta\sin 2\chi(1 + \cos 2\phi)] \\
& + (1/8)\beta_{aaa}[\sin\theta(1 - \cos 2\chi)(\cos\phi - \cos 3\phi) + (\sin\theta - \sin^3\theta)(1 + \cos 2\chi)(3\cos\phi + \cos 3\phi) \\
& \quad - 2\sin\theta\cos\theta\sin 2\chi(\sin\phi + \sin 3\phi)] \\
& + (1/8)\beta_{bba}[\sin\theta(1 - \cos 2\chi)(3\cos\phi + \cos 3\phi) + (\sin\theta - \sin^3\theta)(1 + \cos 2\chi)(\cos\phi - \cos 3\phi) \\
& \quad + 2\sin\theta\cos\theta\sin 2\chi(\sin\phi + \sin 3\phi)] \\
& + (1/2)\beta_{cca}\sin^3\theta(1 + \cos 2\chi)\cos\phi \\
\chi_{zzz} = & \beta_{caa}(\cos\theta - \cos^3\theta)(1 + \cos 2\phi) \\
& + \beta_{bca}(\cos\theta - \cos^3\theta)\sin 2\phi \\
& + (1/4)\beta_{aaa}\sin^3\theta(3\cos\phi + \cos 3\phi) \\
& + (1/4)\beta_{bba}\sin^3\theta(\cos\phi - \cos 3\phi) \\
& + \beta_{cca}(\sin\theta - \sin^3\theta)\cos\phi
\end{aligned}$$

$$\begin{aligned}
(\text{spp}) \quad \chi_{xxx} = & (1/4)\beta_{caa}\{[(\sin\theta - \sin^3\theta)(\sin\chi + \sin 3\chi)(1 + \cos 2\phi) + \sin\theta(\sin\chi - \sin 3\chi)(1 - \cos 2\phi)] \\
& \quad + 2\sin\theta\cos\theta\cos 3\chi\sin 2\phi\} \\
& - (1/4)\beta_{bca}\{2\sin\theta\sin\chi - (2\sin\theta - \sin^3\theta)(\sin\chi + \sin 3\chi)\sin 2\phi + 2\sin\theta\cos\theta(\cos\chi + \cos 3\chi\cos 2\phi)\} \\
& + (1/16)\beta_{aaa}[(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)(3\cos\phi + \cos 3\phi) - 4\cos\theta(\sin\chi\cos\phi + \sin 3\chi\cos 3\phi) \\
& \quad + \sin^2\theta(\cos\chi + 3\cos 3\chi)(\sin\phi + \sin 3\phi) - 4(\cos\chi\sin\phi + \cos 3\chi\sin 3\phi)] \\
& + (1/16)\beta_{bba}[(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)(\cos\phi - \cos 3\phi) + 4\cos\theta(\sin\chi\cos\phi + \sin 3\chi\cos 3\phi) \\
& \quad - \sin^2\theta(\cos\chi - \cos 3\chi)(\sin\phi + \sin 3\phi) + 4\cos^2\theta(\cos\chi\sin\phi + \cos 3\chi\sin 3\phi)] \\
& + (1/4)\beta_{cca}[-(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)\cos\phi + \sin^2\theta(\cos\chi - \cos 3\chi)\sin\phi] \\
\chi_{yzz} = & -(1/2)\beta_{caa}[(\sin\theta - 2\sin^3\theta)\sin\chi(1 + \cos 2\phi) + \sin\theta\cos\theta\cos\chi\sin 2\phi] \\
& - (1/2)\beta_{bca}[(\sin\theta - 2\sin^3\theta)\sin 2\chi\sin 2\phi - \sin\theta\cos\theta\cos\chi(1 + \cos 2\phi)] \\
& - (1/4)\beta_{aaa}[(\cos\theta - \cos^3\theta)\sin\chi(3\cos\phi + \cos 3\phi) + \sin^2\theta\cos\chi(\sin\phi + \sin 3\phi)] \\
& - (1/4)\beta_{bba}[(\cos\theta - \cos^3\theta)\sin\chi(\cos\phi - \cos 3\phi) - \sin^2\theta\cos\chi(\sin\phi + \sin 3\phi)] \\
& + \beta_{cca}(\cos\theta - \cos^3\theta)\sin\chi\cos\phi \\
\chi_{yzz} = & (1/4)\beta_{caa}\{2[(\cos\theta - \cos^3\theta)(1 + \cos 2\phi) - \cos\theta\cos 2\phi]\sin 2\chi + [-\sin^2\theta + (1 - 3\cos^2\theta)\cos 2\chi]\sin 2\phi\} \\
& + (1/4)\beta_{bca}[2\cos^3\theta\sin 2\chi\sin 2\phi - \sin^2\theta(1 - \cos 2\chi)(1 - \cos 2\phi) + 2\cos^2\theta(1 + \cos 2\chi\cos 2\phi)] \\
& + (1/8)\beta_{aaa}[\sin\theta\sin 2\chi(\cos\phi - \cos 3\phi) - (\sin\theta - \sin^3\theta)\sin 2\chi(3\cos\phi + \cos 3\phi)] \\
& - (1/8)\beta_{bba}(2\sin\theta - \sin^3\theta)\sin 2\chi(\cos\phi - \cos 3\phi) \\
& + (1/2)\beta_{cca}[(\sin\theta - \sin^3\theta)\sin 2\chi 3\cos\phi - \sin\theta\cos\theta(1 - \cos 2\chi)\sin\phi] \\
\chi_{yxx} = & (1/2)\beta_{caa}[(\cos\theta - \cos^3\theta)\sin 2\chi(1 + \cos 2\phi) + \sin^2\theta\cos 2\chi\sin 2\phi] \\
& + (1/2)\beta_{bca}[(\cos\theta - \cos^3\theta)\sin 2\chi\sin 2\phi - \sin^2\theta\cos 2\chi(1 + \cos 2\phi)] \\
& + (1/8)\beta_{aaa}[\sin\theta\sin 2\chi(\cos\phi - \cos 3\phi) - (\sin\theta - \sin^3\theta)\sin 2\chi(3\cos\phi + \cos 3\phi) \\
& \quad - 2\sin\theta\cos\theta\cos 2\chi(\sin\phi + \sin 3\phi)] \\
& + (1/8)\beta_{bba}[\sin\theta\sin 2\chi(3\cos\phi + \cos 3\phi) - (\sin\theta - \sin^3\theta)\sin 2\chi(\cos\phi - \cos 3\phi)
\end{aligned}$$

$$+ 2 \sin\theta\cos\theta\cos 2\chi(\sin\phi + \sin 3\phi)] \\ - (1/2)\beta_{cca} \sin^3\theta\sin 2\chi\cos\phi$$

$$(ssp) \quad \chi_{YYX} = (1/4)\beta_{caa} \{ [-(\sin\theta - \sin^3\theta)(1 + \cos 2\phi) + \sin\theta(1 - \cos 2\phi)](\cos\chi - \cos 3\chi) \\ + 2\sin\theta\cos\theta(\sin\chi - \sin 3\chi)\sin 2\phi \} \\ - (1/4)\beta_{bca} [(2\sin\theta - \sin^3\theta)(\cos\chi - \cos 3\chi)]\sin 2\phi - 2\sin 2\theta(\sin\chi + \sin 3\chi)\cos 2\phi \} \\ + (1/16)\beta_{aaa} [-(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)(3\cos\phi + \cos 3\phi) + 4\cos\theta(\cos\chi\cos\phi - \cos 3\chi\cos 3\phi) \\ + \sin^2\theta(\sin\chi - 3\sin 3\chi)(\sin\phi + \sin 3\phi) - (\sin\chi\sin\phi - \sin 3\chi\sin 3\phi)] \\ + (1/16)\beta_{bba} [-(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)(\cos\phi - \cos 3\phi) + 4\cos\theta(3\cos\chi\cos\phi + \cos 3\chi\cos 3\phi) \\ - \sin^2\theta(\sin\chi + \sin 3\chi)(\sin\phi + \sin 3\phi) - \cos^2\theta(3\sin\chi\sin\phi + \sin 3\chi\sin 3\phi)] \\ + (1/4)\beta_{cca} [(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)\cos\phi - \sin^2\theta(3\sin\chi - \sin 3\chi)\sin\phi] \\ \chi_{YYZ} = -(1/2)\beta_{caa} [(\cos\theta - \cos^3\theta)(1 - \cos 2\chi)(1 + \cos 2\phi) + \sin^2\theta\sin 2\chi\sin 2\phi] \\ - (1/2)\beta_{bca} [(\cos\theta - \cos^3\theta)(1 - \cos 2\chi)\sin 2\phi - \sin^2\theta\sin 2\chi(1 + \cos 2\phi)] \\ + (1/8)\beta_{aaa} [\sin\theta(1 + \cos 2\chi)(\cos\phi - \cos 3\phi) + (\sin\theta - \sin^3\theta)(1 - \cos 2\chi)(3\cos\phi + \cos 3\phi) \\ + 2\sin\theta\cos\theta\sin 2\chi(\sin\phi + \sin 3\phi)] \\ + (1/8)\beta_{bba} [\sin\theta(1 + \cos 2\chi)(3\cos\phi + \cos 3\phi) + (\sin\theta - \sin^3\theta)(1 - \cos 2\chi)(\cos\phi - \cos 3\phi) \\ - 2\sin\theta\cos\theta\sin 2\chi(\sin\phi + \sin 3\phi)] \\ + (1/2)\beta_{cca}\sin^3\theta(1 - \cos 2\chi)\cos\phi$$

$$(psp) \quad \chi_{XXY} = (1/4)\beta_{caa} \{ [(\sin\theta - \sin^3\theta)(\sin\chi + \sin 3\chi)(1 + \cos 2\phi) + \sin\theta(\sin\chi - \sin 3\chi)(1 - \cos 2\phi)] \\ + 2\sin\theta\cos\theta\cos 3\chi\sin 2\phi \} \\ - (1/4)\beta_{bca} \{ [2\sin\theta\sin\chi - (2\sin\theta - \sin^3\theta)(\sin\chi + \sin 3\chi)]\sin 2\phi + 2\sin\theta\cos\theta(\cos\chi + \cos 3\chi\cos 2\phi) \} \\ + (1/16)\beta_{aaa} [(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)(3\cos\phi + \cos 3\phi) - 4\cos\theta(\sin\chi\cos\phi + \sin 3\chi\cos 3\phi) \\ + \sin^2\theta(\cos\chi + 3\cos 3\chi)(\sin\phi + \sin 3\phi) - 4(\cos\chi\sin\phi + \cos 3\chi\sin 3\phi)] \\ + (1/16)\beta_{bba} [(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)(\cos\phi - \cos 3\phi) + 4\cos\theta(\sin\chi\cos\phi + \sin 3\chi\cos 3\phi) \\ - \sin^2\theta(\cos\chi - \cos 3\chi)(\sin\phi + \sin 3\phi) + 4\cos^2\theta(\cos\chi\sin\phi + \cos 3\chi\sin 3\phi)] \\ + (1/4)\beta_{cca} [-(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)\cos\phi + \sin^2\theta(\cos\chi - \cos 3\chi)\sin\phi] \\ \chi_{ZZY} = -(1/2)\beta_{caa} [(\sin\theta - 2\sin^3\theta)\sin\chi(1 + \cos 2\phi) + \sin\theta\cos\theta\cos\chi\sin 2\phi] \\ - (1/2)\beta_{bca} [(\sin\theta - 2\sin^3\theta)\sin\chi\sin 2\phi - \sin\theta\cos\theta\cos\chi(1 + \cos 2\phi)] \\ - (1/4)\beta_{aaa} [(\cos\theta - \cos^3\theta)\sin\chi(3\cos\phi + \cos 3\phi) + \sin^2\theta\cos\chi(\sin\phi + \sin 3\phi)] \\ - (1/4)\beta_{bba} [(\cos\theta - \cos^3\theta)\sin\chi(\cos\phi - \cos 3\phi) - \sin^2\theta\cos\chi(\sin\phi + \sin 3\phi)] \\ + \beta_{cca}(\cos\theta - \cos^3\theta)\sin\chi\cos\phi \\ \chi_{XYZ} = (1/2)\beta_{caa} [(\cos\theta - \cos^3\theta)\sin 2\chi(1 + \cos 2\phi) + \sin^2\theta\cos 2\chi\sin 2\phi] \\ + (1/2)\beta_{bca} [(\cos\theta - \cos^3\theta)\sin 2\chi\sin 2\phi - \sin^2\theta\cos 2\chi(1 + \cos 2\phi)] \\ + (1/8)\beta_{aaa} [\sin\theta\sin 2\chi(\cos\phi - \cos 3\phi) - (\sin\theta - \sin^3\theta)\sin 2\chi(3\cos\phi + \cos 3\phi) \\ - 2\sin\theta\cos\theta\cos 2\chi(\sin\phi + \sin 3\phi)] \\ + (1/8)\beta_{bba} [\sin\theta\sin 2\chi(3\cos\phi + \cos 3\phi) - (\sin\theta - \sin^3\theta)\sin 2\chi(\cos\phi - \cos 3\phi) \\ + 2\sin\theta\cos\theta\cos 2\chi(\sin\phi + \sin 3\phi)] \\ - (1/2)\beta_{cca}\sin^3\theta\sin 2\chi\cos\phi \\ \chi_{ZXY} = (1/4)\beta_{caa} \{ 2[(\cos\theta - \cos^3\theta)(1 + \cos 2\phi) - \cos\theta\cos 2\phi]\sin 2\chi + [-\sin^2\theta + (1 - 3\cos^2\theta)\cos 2\chi]\sin 2\phi \} \\ + (1/4)\beta_{bca} [2\cos^3\theta\sin 2\chi\sin 2\phi - \sin^2\theta(1 - \cos 2\chi)(1 - \cos 2\phi) + 2\cos^2\theta(1 + \cos 2\chi\cos 2\phi)]$$

$$\begin{aligned}
& + (1/8)\beta_{aaa}[\sin\theta\sin 2\chi(\cos\phi - \cos 3\phi) - (\sin\theta - \sin^3\theta)\sin 2\chi(3\cos\phi + \cos 3\phi) \\
& \quad - 2\sin\theta\cos\theta\cos 2\chi(\sin\phi + \sin 3\phi)] \\
& + (1/8)\beta_{bba}[\sin\theta\sin 2\chi(3\cos\phi + \cos 3\phi) - (\sin\theta - \sin^3\theta)\sin 2\chi(\cos\phi - \cos 3\phi) \\
& \quad + 2\sin\theta\cos\theta\cos 2\chi(\sin\phi + \sin 3\phi)] \\
& + (1/2)\beta_{caa}[(\sin\theta - \sin^3\theta)\sin 2\chi\cos\phi - \sin\theta\cos\theta(1 - \cos 2\chi)\sin\phi]
\end{aligned}$$

$$\begin{aligned}
(\text{sps}) \quad \chi_{XXY} &= -(1/4)\beta_{caa}\{[(\sin\theta - \sin^3\theta)(\cos\chi - \cos 3\chi)(1 + \cos 2\phi) + \sin\theta(\cos\chi + \cos 3\chi)(1 - \cos 2\phi)] \\
& \quad + 2\sin\theta\cos\theta\sin 3\chi\sin 2\phi\} \\
& + (1/4)\beta_{bca}\{2\sin\theta\cos\chi - (\sin\theta - \sin^3\theta)(\cos\chi - \cos 3\chi)\sin 2\phi - 2\sin\theta\cos\theta(\sin\chi - \sin 3\chi\cos 2\phi)\} \\
& + (1/16)\beta_{aaa}\{-(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)(3\cos\phi + \cos 3\phi) + 4\cos\theta(\cos\chi\cos\phi - \cos 3\chi\cos 3\phi) \\
& \quad + \sin^2\theta(\sin\chi - 3\sin 3\chi)(\sin\phi + \sin 3\phi) - 4(\sin\chi\sin\phi - \sin 3\chi\sin 3\phi)\} \\
& + (1/16)\beta_{bba}\{-(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)(\cos\phi - \cos 3\phi) - 4\cos\theta(\cos\chi\cos\phi - \cos 3\chi\cos 3\phi) \\
& \quad - \sin^2\theta(\sin\chi + \sin 3\chi)(\sin\phi + \sin 3\phi) + 4\cos^2\theta(\sin\chi\sin\phi - \sin 3\chi\sin 3\phi)\} \\
& + (1/4)\beta_{caa}\{(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)\cos\phi + \sin^2\theta(\sin\chi + \sin 3\chi)\sin\phi\} \\
\chi_{YZY} &= (1/4)\beta_{caa}\{2[\cos\theta(1 - \cos 2\phi\cos 2\chi) - (\cos\theta - \cos^3\theta)(1 - \cos 2\chi)(1 + \cos 2\phi)] \\
& \quad - (1 - 3\cos^2\theta)\sin 2\chi\sin 2\phi\} \\
& - (1/4)\beta_{bca}\{2[(\cos\theta - \cos^3\theta) - \cos^3\theta\cos 2\chi]\sin 2\phi + [\sin^2\theta - (1 - 3\cos^2\theta)\cos 2\phi]\sin 2\chi\} \\
& + (1/8)\beta_{aaa}\{\sin\theta(1 + \cos 2\chi)(\cos\phi - \cos 3\phi) + (\sin\theta - \sin^3\theta)(1 - \cos 2\chi)(3\cos\phi + \cos 3\phi) \\
& \quad + 2\sin\theta\cos\theta\sin 2\chi(\sin\phi + \sin 3\phi)\} \\
& + (1/8)\beta_{bba}\{-\sin\theta(1 + \cos 2\chi)(\cos\phi - \cos 3\phi) + (\sin\theta - \sin^3\theta)(1 - \cos 2\chi)(\cos\phi - \cos 3\phi) \\
& \quad + 2\sin\theta\cos\theta\sin 2\chi(\sin\phi - \sin 3\phi)\} \\
& - (1/2)\beta_{caa}\{(\sin\theta - \sin^3\theta)(1 - \cos 2\chi)\cos\phi - \sin\theta\cos\theta\sin 2\chi\sin\phi\}
\end{aligned}$$

$$\begin{aligned}
(\text{pps}) \quad \chi_{XXY} &= (1/4)\beta_{caa}\{[(\sin\theta - \sin^3\theta)(1 + \cos 2\phi) - \sin\theta(1 - \cos 2\phi)](\sin\chi + \sin 3\chi) \\
& \quad + 2\sin\theta\cos\theta(\cos\chi + \cos 3\chi)\sin 2\phi\} \\
& + (1/4)\beta_{bca}\{2\sin\theta - \sin^3\theta)(\sin\chi + \sin 3\chi)\sin 2\phi + 2\sin\theta\cos\theta(\cos\chi - \cos 3\chi)\cos 2\phi\} \\
& + (1/16)\beta_{aaa}\{(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)(3\cos\phi + \cos 3\phi) - 4\cos\theta(\sin\chi\cos\phi + \sin 3\chi\cos 3\phi) \\
& \quad + \sin^2\theta(\cos\chi + 3\cos 3\chi)(\sin\phi + \sin 3\phi) - 4(\cos\chi\sin\phi + \cos 3\chi\sin 3\phi)\} \\
& + (1/16)\beta_{bba}\{(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)(\cos\phi - \cos 3\phi) - 4\cos\theta(3\sin\chi\cos\phi - \sin 3\chi\cos 3\phi) \\
& \quad - \sin^2\theta(\cos\chi - \cos 3\chi)(\sin\phi + \sin 3\phi) - 4\cos^2\theta(3\cos\chi\sin\phi - \cos 3\chi\sin 3\phi)\} \\
& - (1/4)\beta_{caa}\{(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)\cos\phi + \sin^2\theta(3\cos\chi + \cos 3\chi)\sin\phi\} \\
\chi_{ZZY} &= -\beta_{caa}\{(\sin\theta - \sin^3\theta)\sin\chi(1 + \cos 2\phi) + \sin\theta\cos\theta\cos\chi\sin 2\phi\} \\
& \quad - \beta_{bca}\{(\sin\theta - \sin^3\theta)\sin\chi\sin 2\phi + \sin\theta\cos\theta\cos\chi(1 - \cos 2\phi)\} \\
& \quad - (1/4)\beta_{aaa}\{(\cos\theta - \cos^3\theta)\sin\chi(3\cos\phi + \cos 3\phi) + \sin^2\theta\cos\chi(\sin\phi + \sin 3\phi)\} \\
& \quad - (1/4)\beta_{bba}\{(\cos\theta - \cos^3\theta)\sin\chi(\cos\phi - \cos 3\phi) + \sin^2\theta\cos\chi(3\sin\phi - \sin 3\phi)\} \\
& \quad - \beta_{caa}\{\cos^3\theta\sin\chi\cos\phi + \cos^2\theta\cos\chi\sin\phi\} \\
\chi_{ZZY} &= (1/4)\beta_{caa}\{2[(\cos\theta - \cos^3\theta)(1 + \cos 2\phi) - \cos\theta\cos 2\phi]\sin 2\chi + [\sin^2\theta + (1 - 3\cos^2\theta)\cos 2\chi]\sin 2\phi\} \\
& + (1/4)\beta_{bca}\{2\cos^3\theta\sin 2\chi\sin 2\phi + \sin^2\theta(1 + \cos 2\chi)(1 - \cos 2\phi) - 2\cos^2\theta(1 - \cos 2\chi\cos 2\phi)\} \\
& - (1/4)\beta_{aaa}\{\sin\theta\sin 2\chi(\cos\phi - \cos 3\phi) - (\sin\theta - \sin^3\theta)\sin 2\chi(3\cos\phi + \cos 3\phi) \\
& \quad - 2\sin\theta\cos\theta\cos 2\chi(\sin\phi + \sin 3\phi)\} \\
& - (1/4)\beta_{bba}\{\sin\theta\sin 2\chi(\cos\phi - \cos 3\phi) + (\sin\theta - \sin^3\theta)\sin 2\chi(\cos\phi - \cos 3\phi)
\end{aligned}$$

$$\begin{aligned}
& + 2\sin\theta\cos\theta[2\sin\phi + \cos 2\chi(\sin\phi - \sin 3\phi)] \} \\
\chi_{XZY} = & (1/2)\beta_{cca}[(\sin\theta - \sin^3\theta)\sin 2\chi\cos\phi + \sin\theta\cos\theta(1 + \cos 2\chi)\sin\phi] \\
& + (1/4)\beta_{caa}\{ 2[(\cos\theta - \cos^3\theta)(1 + \cos 2\phi) - \cos\theta\cos 2\phi]\sin 2\chi + [\sin^2\theta + (1 - 3\cos^2\theta)\cos 2\chi]\sin 2\phi \} \\
& + (1/4)\beta_{bca}[2\cos^3\theta\sin 2\chi\sin 2\phi + \sin^2\theta(1 + \cos 2\chi)(1 - \cos 2\phi) - 2\cos^2\theta(1 - \cos 2\chi\cos 2\phi)] \\
& + (1/4)\beta_{aaa}[\sin\theta\sin 2\chi(\cos\phi - \cos 3\phi) - (\sin\theta - \sin^3\theta)\sin 2\chi(3\cos\phi + \cos 3\phi) \\
& \quad - 2\sin\theta\cos\theta\cos 2\chi(\sin\phi + \sin 3\phi)] \\
& - (1/4)\beta_{bba}\{ \sin\theta\sin 2\chi(\cos\phi - \cos 3\phi) + (\sin\theta - \sin^3\theta)\sin 2\chi(\cos\phi - \cos 3\phi) \\
& \quad + 2\sin\theta\cos\theta[2\sin\phi + \cos 2\chi(\sin\phi - \sin 3\phi)] \} \\
& + (1/2)\beta_{cca}[(\sin\theta - \sin^3\theta)\sin 2\chi\cos\phi + \sin\theta\cos\theta(1 + \cos 2\chi)\sin\phi] \\
\text{(pss)} \quad \chi_{XYX} = & -(1/4)\beta_{caa}\{ [(\sin\theta - \sin^3\theta)(\cos\chi - \cos 3\chi)(1 + \cos 2\phi) + \sin\theta(\cos\chi + \cos 3\chi)(1 - \cos 2\phi)] \\
& \quad + 2\sin\theta\cos\theta\sin 3\chi\sin 2\phi \} \\
& + (1/4)\beta_{bca}\{ [2\sin\theta\cos\chi - (2\sin\theta - \sin^3\theta)(\cos\chi - \cos 3\chi)]\sin 2\phi - 2\sin\theta\cos\theta(\sin\chi - \sin 3\chi\cos 2\phi) \} \\
& + (1/16)\beta_{aaa}[-(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)(3\cos\phi + \cos 3\phi) + 4\cos\theta(\cos\chi\cos\phi - \cos 3\chi\cos 3\phi) \\
& \quad + \sin^2\theta(\sin\chi - 3\sin 3\chi)(\sin\phi + \sin 3\phi) - 4(\sin\chi\sin\phi - \sin 3\chi\sin 3\phi)] \\
& + (1/16)\beta_{bba}[-(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)(\cos\phi - \cos 3\phi) - 4\cos\theta(\cos\chi\cos\phi - \cos 3\chi\cos 3\phi) \\
& \quad - \sin^2\theta(\sin\chi + \sin 3\chi)(\sin\phi + \sin 3\phi) + 4\cos^2\theta(\sin\chi\sin\phi - \sin 3\chi\sin 3\phi)] \\
& + (1/4)\beta_{cca}[(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)\cos\phi + \sin^2\theta(\sin\chi + \sin 3\chi)\sin\phi] \\
\chi_{ZYX} = & (1/4)\beta_{caa}\{ 2[\cos\theta(1 - \cos 2\phi\cos 2\chi) - (\cos\theta - \cos^3\theta)(1 - \cos 2\chi)(1 + \cos 2\phi)] \\
& \quad - (1 - 3\cos^2\theta)\sin 2\chi\sin 2\phi \} \\
& - (1/4)\beta_{bca}\{ 2[(\cos\theta - \cos^3\theta) - \cos^3\theta\cos 2\chi]\sin 2\phi + [\sin^2\theta - (1 - 3\cos^2\theta)\cos 2\phi]\sin 2\chi \} \\
& + (1/8)\beta_{aaa}[\sin\theta(1 + \cos 2\chi)(\cos\phi - \cos 3\phi) + (\sin\theta - \sin^3\theta)(1 - \cos 2\chi)(3\cos\phi + \cos 3\phi) \\
& \quad + 2\sin\theta\cos\theta\sin 2\chi(\sin\phi + \sin 3\phi)] \\
& + (1/8)\beta_{bba}[-\sin\theta(1 + \cos 2\chi)(\cos\phi - \cos 3\phi) + (\sin\theta - \sin^3\theta)(1 - \cos 2\chi)(\cos\phi - \cos 3\phi) \\
& \quad + 2\sin\theta\cos\theta\sin 2\chi(\sin\phi - \sin 3\phi)] \\
& - (1/2)\beta_{caa}[(\sin\theta - \sin^3\theta)(1 + \cos 2\chi)\cos\phi + \sin\theta\cos\theta\sin 2\chi\sin\phi] \\
\text{(sss)} \quad \chi_{YYX} = & (1/4)\beta_{caa}\{ [(\sin\theta - \sin^3\theta)(3\sin\chi - \sin 3\chi)(1 + \cos 2\phi) + \sin\theta(\sin\chi + \sin 3\chi)(1 - \cos 2\phi)] \\
& \quad + 2\sin\theta\cos\theta(\cos\chi - \cos 3\chi)\sin 2\phi \} \\
& + (1/4)\beta_{bca}\{ [4(\sin\theta - \sin^3\theta)\sin\chi - (2\sin\theta - \sin^3\theta)(\sin\chi + \sin 3\chi)]\sin 2\phi \\
& \quad - 2\sin\theta\cos\theta(\cos\chi - \cos 3\chi)\cos 2\phi \} \\
& + (1/16)\beta_{aaa}[(\cos\theta - \cos^3\theta)(3\sin\chi - \sin 3\chi)(3\cos\phi + \cos 3\phi) - 4\cos\theta(3\sin\chi\cos\phi - \sin 3\chi\cos 3\phi) \\
& \quad + 3\sin^2\theta(\cos\chi - \cos 3\chi)(\sin\phi + \sin 3\phi) - 4(3\cos\chi\sin\phi - \cos 3\chi\sin 3\phi)] \\
& + (1/16)\beta_{bba}[(\cos\theta - \cos^3\theta)(3\sin\chi - \sin 3\chi)(\cos\phi - \cos 3\phi) - 4\cos\theta(\sin\chi\cos\phi + \sin 3\chi\cos 3\phi) \\
& \quad + \sin^2\theta(\cos\chi - \cos 3\chi)(\sin\phi - \sin 3\phi) - 4(\cos\chi\sin\phi + \cos 3\chi\sin 3\phi)] \\
& - (1/4)\beta_{caa}[(\cos\theta - \cos^3\theta)(3\sin\chi - \sin 3\chi)\cos\phi + \sin^2\theta(\cos\chi - \cos 3\chi)\sin\phi] \\
\text{(ppp)} \quad \chi_{XXX} = & -(1/4)\beta_{cbb}\{ [\sin\theta(\cos\chi - \cos 3\chi)(1 + \cos 2\phi) + (\sin\theta - \sin^3\theta)(3\cos\chi + \cos 3\chi)(1 - \cos 2\phi)] \\
& \quad + 2\sin\theta\cos\theta(\sin\chi + \sin 3\chi)\sin 2\phi \} \\
& + (1/4)\beta_{cab}\{ [\sin\theta(\cos\chi - \cos 3\chi) - (\sin\theta - \sin^3\theta)(3\cos\chi + \cos 3\chi)]\sin 2\phi \\
& \quad - 2\sin\theta\cos\theta(\sin\chi + \sin 3\chi)\cos 2\phi \} \\
& + (1/16)\beta_{aab}[-(\cos\theta - \cos^3\theta)(3\cos\chi + \cos 3\chi)(\sin\phi + \sin 3\phi) + 4\cos\theta(\cos\chi\sin\phi + \cos 3\chi\sin 3\phi)]
\end{aligned}$$

$$\begin{aligned}
& - \sin^2\theta(\sin\chi + \sin 3\chi)(\cos\phi + \cos 3\phi) + 4(\sin\chi\cos\phi + \sin 3\chi\cos 3\phi)] \\
& + (1/16)\beta_{bbb}[-(\cos\theta - \cos^3\theta)(3\cos\chi + \cos 3\chi)(3\sin\phi - \sin 3\phi) + 4\cos\theta(3\cos\chi\sin\phi - \cos 3\chi\sin 3\phi) \\
& \quad - 3\sin^2\theta(\sin\chi + \sin 3\chi)(\cos\phi - \cos 3\phi) + 4(3\sin\chi\cos\phi - \sin 3\chi\cos 3\phi)] \\
& + (1/4)\beta_{c_{cb}}[(\cos\theta - \cos^3\theta)(3\cos\chi + \cos 3\chi)\sin\phi + \sin^2\theta(\sin\chi + \sin 3\chi)\cos\phi] \\
& + (1/8)\beta_{abb}[-(\cos\theta - \cos^3\theta)(3\cos\chi + \cos 3\chi)(\cos\phi - \cos 3\phi) + 4\cos\theta(\cos\chi\cos\phi - \cos 3\chi\cos 3\phi) \\
& \quad + 3\sin^2\theta(\sin\chi + \sin 3\chi)(\sin\phi - \sin 3\phi) - 4(\sin\chi\sin\phi - \sin 3\chi\sin 3\phi)] \\
\chi_{ZZZ} = & (1/2)\beta_{c_{bb}}[(\sin\theta - 2\sin^3\theta)\cos\chi(1 - \cos 2\phi) + \sin\theta\cos\theta\sin\chi\sin 2\phi] \\
& + (1/2)\beta_{c_{cb}}[(\sin\theta - 2\sin^3\theta)\cos\chi\sin 2\phi - \sin\theta\cos\theta\sin\chi(1 - \cos 2\phi)] \\
& + (1/4)\beta_{a_{ab}}[(\cos\theta - \cos^3\theta)\cos\chi(\sin\phi + \sin 3\phi) - \sin^2\theta\sin\chi(\cos\phi - \cos 3\phi)] \\
& + (1/4)\beta_{bbb}[(\cos\theta - \cos^3\theta)\cos\chi(3\sin\phi - \sin 3\phi) + \sin^2\theta\sin\chi(\cos\phi - \cos 3\phi)] \\
& + \beta_{c_{cb}}(\cos\theta - \cos^3\theta)\cos\chi\sin\phi \\
& + (1/2)\beta_{abb}[(\cos\theta - \cos^3\theta)\cos\chi(\cos\phi - \cos 3\phi) - \sin^2\theta\sin\chi(\sin\phi - \sin 3\phi)] \\
\chi_{ZZX} = & (1/2)\beta_{c_{bb}}[(\sin\theta - 2\sin^3\theta)\cos\chi(1 - \cos 2\phi) + \sin\theta\cos\theta\sin\chi\sin 2\phi] \\
& + (1/2)\beta_{c_{cb}}[(\sin\theta - 2\sin^3\theta)\cos\chi\sin 2\phi - \sin\theta\cos\theta\sin\chi(1 - \cos 2\phi)] \\
& + (1/4)\beta_{a_{ab}}[(\cos\theta - \cos^3\theta)\cos\chi(\sin\phi + \sin 3\phi) - \sin^2\theta\sin\chi(\cos\phi - \cos 3\phi)] \\
& + (1/4)\beta_{bbb}[(\cos\theta - \cos^3\theta)\cos\chi(3\sin\phi - \sin 3\phi) + \sin^2\theta\sin\chi(\cos\phi - \cos 3\phi)] \\
& + \beta_{c_{cb}}(\cos\theta - \cos^3\theta)\cos\chi\sin\phi \\
& + (1/2)\beta_{abb}[(\cos\theta - \cos^3\theta)\cos\chi(\cos\phi - \cos 3\phi) - \sin^2\theta\sin\chi(\sin\phi - \sin 3\phi)] \\
\chi_{ZZX} = & \beta_{c_{bb}}[(\sin\theta - \sin^3\theta)\cos\chi(1 - \cos 2\phi) + \sin\theta\cos\theta\sin\chi\sin 2\phi] \\
& + \beta_{c_{cb}}[(\sin\theta - \sin^3\theta)\cos\chi\sin 2\phi + \sin\theta\cos\theta\sin\chi(1 + \cos 2\phi)] \\
& + (1/4)\beta_{a_{ab}}[(\cos\theta - \cos^3\theta)\cos\chi(\sin\phi + \sin 3\phi) + \sin^2\theta\sin\chi(3\cos\phi + \cos 3\phi)] \\
& + (1/4)\beta_{bbb}[(\cos\theta - \cos^3\theta)\cos\chi(3\sin\phi - \sin 3\phi) + \sin^2\theta\sin\chi(\cos\phi - \cos 3\phi)] \\
& - \beta_{c_{cb}}\cos^3\theta\cos\chi\sin\phi \\
& + (1/2)\beta_{abb}[(\cos\theta - \cos^3\theta)\cos\chi(\cos\phi - \cos 3\phi) + \sin^2\theta\sin\chi(\sin\phi + \sin 3\phi)] \\
\chi_{ZXX} = & (1/4)\beta_{c_{bb}}\{2[\cos\theta(1 - \cos 2\phi\cos 2\chi) - (\cos\theta - \cos^3\theta)(1 + \cos 2\chi)(1 - \cos 2\phi)] \\
& \quad - (1 - 3\cos^2\theta)\sin 2\chi\sin 2\phi\} \\
& - (1/4)\beta_{c_{cb}}\{2[(\cos\theta - \cos^3\theta) - \cos^3\theta\cos 2\chi]\sin 2\phi + [\sin^2\theta + (1 - 3\cos^2\theta)\cos 2\phi]\sin 2\chi\} \\
& + (1/8)\beta_{a_{ab}}[(\sin\theta - \sin^3\theta)(1 + \cos 2\chi)(\sin\phi + \sin 3\phi) + \sin\theta(1 - \cos 2\chi)(\sin\phi + \sin 3\phi) \\
& \quad + 2\sin\theta\cos\theta\sin 2\chi(\cos\phi + \cos 3\phi)] \\
& + (1/8)\beta_{bbb}[(\sin\theta - \sin^3\theta)(1 + \cos 2\chi)(3\sin\phi - \sin 3\phi) - \sin\theta(1 - \cos 2\chi)(\sin\phi + \sin 3\phi) \\
& \quad + 2\sin\theta\cos\theta\sin 2\chi(\cos\phi - \cos 3\phi)] \\
& - (1/2)\beta_{c_{cb}}[(\sin\theta - \sin^3\theta)(1 + \cos 2\chi)\sin\phi + \sin\theta\cos\theta\sin 2\chi\cos\phi] \\
& + (1/4)\beta_{abb}[(\sin\theta - \sin^3\theta)(1 + \cos 2\chi)(\cos\phi - \cos 3\phi) + \sin\theta(1 - \cos 2\chi)(\cos\phi + \cos 3\phi) \\
& \quad + 2\sin\theta\cos\theta\sin 2\chi\sin 3\phi] \\
\chi_{XZX} = & (1/4)\beta_{c_{bb}}\{2[\cos\theta(1 - \cos 2\phi\cos 2\chi) - (\cos\theta - \cos^3\theta)(1 + \cos 2\chi)(1 - \cos 2\phi)] \\
& \quad - (1 - 3\cos^2\theta)\sin 2\chi\sin 2\phi\} \\
& - (1/4)\beta_{c_{cb}}\{2[(\cos\theta - \cos^3\theta) - \cos^3\theta\cos 2\chi]\sin 2\phi + [\sin^2\theta + (1 - 3\cos^2\theta)\cos 2\phi]\sin 2\chi\} \\
& + (1/8)\beta_{a_{ab}}[(\sin\theta - \sin^3\theta)(1 + \cos 2\chi)(\sin\phi + \sin 3\phi) - \sin\theta(1 - \cos 2\chi)(\sin\phi + \sin 3\phi) \\
& \quad + 2\sin\theta\cos\theta\sin 2\chi(\cos\phi + \cos 3\phi)] \\
& + (1/8)\beta_{bbb}[(\sin\theta - \sin^3\theta)(1 + \cos 2\chi)(3\sin\phi - \sin 3\phi) + \sin\theta(1 - \cos 2\chi)(\sin\phi + \sin 3\phi) \\
& \quad + 2\sin\theta\cos\theta\sin 2\chi(\cos\phi - \cos 3\phi)]
\end{aligned}$$

$$\begin{aligned}
& - (1/2)\beta_{cb}[(\sin\theta - \sin^3\theta)(1 + \cos 2\chi)\sin\phi + \sin\theta\cos\theta\sin 2\chi\cos\phi] \\
& + (1/4)\beta_{abb}[(\sin\theta - \sin^3\theta)(1 + \cos 2\chi)(\cos\phi - \cos 3\phi) + \sin\theta(1 - \cos 2\chi)(\cos\phi + \cos 3\phi) \\
& \quad + 2\sin\theta\cos\theta\sin 2\chi\sin 3\phi] \\
\chi_{XXZ} = & - (1/2)\beta_{cbb}[(\cos\theta - \cos^3\theta)(1 + \cos 2\chi)(1 - \cos 2\phi) + \sin^2\theta\sin 2\chi\sin 2\phi] \\
& - (1/2)\beta_{cab}[(\cos\theta - \cos^3\theta)(1 + \cos 2\chi)\sin 2\phi - \sin^2\theta\sin 2\chi(1 - \cos 2\phi)] \\
& + (1/8)\beta_{aab}[(\sin\theta - \sin^3\theta)(1 + \cos 2\chi)(\sin\phi + \sin 3\phi) + \sin\theta(1 - \cos 2\chi)(3\sin\phi - \sin 3\phi) \\
& \quad - 2\sin\theta\cos\theta\sin 2\chi(\cos\phi - \cos 3\phi)] \\
& + (1/8)\beta_{bbb}[(\sin\theta - \sin^3\theta)(1 + \cos 2\chi)(3\sin\phi - \sin 3\phi) + \sin\theta(1 - \cos 2\chi)(\sin\phi + \sin 3\phi) \\
& \quad + 2\sin\theta\cos\theta\sin 2\chi(\cos\phi - \cos 3\phi)] \\
& + (1/2)\beta_{cbb}\sin^3\theta(1 + \cos 2\chi)\sin\phi \\
& + (1/4)\beta_{abb}[(\sin\theta - \sin^3\theta)(1 + \cos 2\chi)(\cos\phi - \cos 3\phi) - \sin\theta(1 - \cos 2\chi)(\cos\phi - \cos 3\phi) \\
& \quad - 2\sin\theta\cos\theta\sin 2\chi(\sin\phi - \sin 3\phi)] \\
\chi_{ZZZ} = & \beta_{cbb}(\cos\theta - \cos^3\theta)(1 - \cos 2\phi) \\
& + \beta_{cab}(\cos\theta - \cos^3\theta)\sin 2\phi \\
& + (1/4)\beta_{aab}\sin^3\theta(\sin\phi + \sin 3\phi) \\
& + (1/4)\beta_{bbb}\sin^3\theta(3\sin\phi - \sin 3\phi) \\
& + \beta_{cbb}(\sin\theta - \sin^3\theta)\sin\phi \\
& + (1/2)\beta_{abb}\sin^3\theta(\cos\phi - \cos 3\phi)
\end{aligned}$$

$$\begin{aligned}
(\text{spp}) \quad \chi_{YXX} = & (1/4)\beta_{cbb}\{[\sin\theta(\sin\chi - \sin 3\chi)(1 + \cos 2\phi) + (\sin\theta - \sin^3\theta)(\sin\chi + \sin 3\chi)(1 - \cos 2\phi)] \\
& \quad - 2\sin\theta\cos\theta\cos 3\chi\sin 2\phi\} \\
& - (1/4)\beta_{cab}\{[\sin\theta(\sin\chi - \sin 3\chi) + (\sin\theta - \sin^3\theta)(\sin\chi + \sin 3\chi)]\sin 2\phi \\
& \quad - 2\sin\theta\cos\theta(\cos\chi - \cos 3\chi)\cos 2\phi\} \\
& + (1/16)\beta_{aab}[(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)(\sin\phi + \sin 3\phi) + 4\cos\theta(\sin\chi\sin\phi - \sin 3\chi\sin 3\phi) \\
& \quad + \sin^2\theta(\cos\chi - \cos 3\chi)(\cos\phi - \cos 3\phi) - 4\cos^2\theta(\cos\chi\cos\phi - \cos 3\chi\cos 3\phi)] \\
& + (1/16)\beta_{bbb}[(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)(3\sin\phi - \sin 3\phi) - 4\cos\theta(\sin\chi\sin\phi - \sin 3\chi\sin 3\phi) \\
& \quad - \sin^2\theta(\cos\chi + 3\cos 3\chi)(\cos\phi - \cos 3\phi) + 4(\cos\chi\cos\phi - \cos 3\chi\cos 3\phi)] \\
& - (1/4)\beta_{cbb}[(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)\sin\phi + \sin^2\theta(\cos\chi - \cos 3\chi)\cos\phi] \\
& + (1/8)\beta_{abb}[(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)(\cos\phi - \cos 3\phi) - 4\cos\theta(\sin\chi\cos\phi - \sin 3\chi\cos 3\phi) \\
& \quad - \sin^2\theta(\cos\chi - \cos 3\chi)(\sin\phi + \sin 3\phi) - 4\cos^2\theta(\cos\chi\sin\phi - \cos 3\chi\sin 3\phi)] \\
\chi_{YZZ} = & - (1/2)\beta_{cbb}[(\sin\theta - 2\sin^3\theta)\sin\chi(1 - \cos 2\phi) - \sin\theta\cos\theta\cos\chi\sin 2\phi] \\
& - (1/2)\beta_{cab}[(\sin\theta - 2\sin^3\theta)\sin\chi\sin 2\phi + \sin\theta\cos\theta\cos\chi(1 - \cos 2\phi)] \\
& - (1/4)\beta_{aab}[(\cos\theta - \cos^3\theta)\sin\chi(\sin\phi + \sin 3\phi) + \sin^2\theta\cos\chi(\cos\phi - \cos 3\phi)] \\
& - (1/4)\beta_{bbb}[(\cos\theta - \cos^3\theta)\sin\chi(3\sin\phi - \sin 3\phi) - \sin^2\theta\cos\chi(\cos\phi - \cos 3\phi)] \\
& + \beta_{cbb}(\cos\theta - \cos^3\theta)\sin\chi\sin\phi \\
& - (1/2)\beta_{abb}[(\cos\theta - \cos^3\theta)\sin\chi(\cos\phi - \cos 3\phi) + \sin^2\theta\cos\chi(\sin\phi - \sin 3\phi)] \\
\chi_{YZX} = & (1/4)\beta_{cbb}\{2[(\cos\theta - \cos^3\theta)(1 - \cos 2\phi) + \cos\theta\cos 2\phi]\sin 2\chi + [\sin^2\theta - (1 - 3\cos^2\theta)\cos 2\chi]\sin 2\phi\} \\
& - (1/4)\beta_{cab}\{2\cos^3\theta\sin 2\chi\sin 2\phi - \sin^2\theta(1 - \cos 2\chi)(1 + \cos 2\phi) + 2\cos^2\theta(1 - \cos 2\chi)\cos 2\phi\} \\
& - (1/8)\beta_{aab}\{2(\sin\theta - \sin^3\theta)\sin 2\chi(\sin\phi + \sin 3\phi) + 2\sin\theta\cos\theta[2\cos\phi - \cos 2\chi(\cos\phi - \cos 3\phi)]\} \\
& + (1/8)\beta_{bbb}\{-(\sin\theta - \sin^3\theta)\sin 2\chi(3\sin\phi - \sin 3\phi) + \sin\theta\sin 2\chi(\sin\phi + \sin 3\phi) \\
& \quad + 2\sin\theta\cos\theta\cos 2\chi(\cos\phi - \cos 3\phi)\}
\end{aligned}$$

$$\begin{aligned}
& + (1/2)\beta_{cbb}[(\sin\theta - \sin^3\theta)\sin 2\chi\sin\phi + \sin\theta\cos\theta(1 - \cos 2\chi)\cos\phi] \\
& + (1/4)\beta_{abb}[-(\sin\theta - \sin^3\theta)\sin 2\chi(\cos\phi - \cos 3\phi) + \sin\theta\sin 2\chi(\cos\phi + \cos 3\phi) \\
& \quad - 2\sin\theta\cos\theta(\sin\phi + \cos 2\chi\sin 3\phi)] \\
\chi_{YXZ} = & (1/2)\beta_{cbb}[(\cos\theta - \cos^3\theta)\sin 2\chi(1 - \cos 2\phi) - \sin^2\theta\cos 2\chi\sin 2\phi] \\
& + (1/2)\beta_{cab}[(\cos\theta - \cos^3\theta)\sin 2\chi\sin 2\phi + \sin^2\theta\cos 2\chi(1 - \cos 2\phi)] \\
& + (1/8)\beta_{aab}[-(\sin\theta - \sin^3\theta)\sin 2\chi(\sin\phi + \sin 3\phi) + \sin\theta\sin 2\chi(3\sin\phi - \sin 3\phi) \\
& \quad - 2\sin\theta\cos\theta\cos 2\chi(\cos\phi - \cos 3\phi)] \\
& + (1/8)\beta_{bbb}[-(\sin\theta - \sin^3\theta)\sin 2\chi(3\sin\phi - \sin 3\phi) + \sin\theta\sin 2\chi(\sin\phi + \sin 3\phi) \\
& \quad + 2\sin\theta\cos\theta\cos 2\chi(\cos\phi - \cos 3\phi)] \\
& - (1/2)\beta_{cbb}\sin^3\theta\sin 2\chi\sin\phi \\
& - (1/4)\beta_{abb}[(2\sin\theta - \sin^3\theta)\sin 2\chi(\cos\phi - \cos 3\phi) + 2\sin\theta\cos\theta\cos 2\chi(\sin\phi - \sin 3\phi)]
\end{aligned}$$

$$\begin{aligned}
(\text{ssp}) \quad \chi_{YYX} = & (1/4)\beta_{cbb}\{[\sin\theta(1 + \cos 2\phi) - (\sin\theta - \sin^3\theta)(1 - \cos 2\phi)](\cos\chi - \cos 3\chi) \\
& \quad - 2\sin\theta\cos\theta(\sin\chi - \sin 3\chi)\sin 2\phi\} \\
& - (1/4)\beta_{cab}\{ (2\sin\theta - \sin^3\theta)(\cos\chi - \cos 3\chi)\sin 2\phi + 2\sin\theta\cos\theta[2\sin\chi - (\sin\chi - \sin 3\chi)\cos 2\phi]\} \\
& + (1/16)\beta_{aab}[-(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)(\sin\phi + \sin 3\phi) + 4\cos\theta(3\cos\chi\sin\phi - \cos 3\chi\sin 3\phi) \\
& \quad + \sin^2\theta(\sin\chi + \sin 3\chi)(\cos\phi + \cos 3\phi) + 4\cos^2\theta(3\sin\chi\cos\phi - \sin 3\chi\cos 3\phi)] \\
& + (1/16)\beta_{bbb}[-(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)(3\sin\phi - \sin 3\phi) + 4\cos\theta(\cos\chi\sin\phi + \cos 3\chi\sin 3\phi) \\
& \quad - \sin^2\theta(\sin\chi - 3\sin 3\chi)(\cos\phi - \cos 3\phi) + 4\cos^2\theta(\sin\chi\cos\phi + \sin 3\chi\cos 3\phi)] \\
& + (1/4)\beta_{cbb}[(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)\sin\phi + \sin^2\theta(3\sin\chi - \sin 3\chi)\cos\phi] \\
& + (1/8)\beta_{abb}[-(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)(\cos\phi - \cos 3\phi) - 4\cos\theta(\cos\chi\cos\phi - \cos 3\chi\cos 3\phi) \\
& \quad - \sin^2\theta(\sin\chi + \sin 3\chi)(\sin\phi + \sin 3\phi) + 4\cos^2\theta(\sin\chi\sin\phi - \sin 3\chi\sin 3\phi)] \\
\chi_{YYZ} = & -(1/2)\beta_{cbb}[(\cos\theta - \cos^3\theta)(1 - \cos 2\chi)(1 - \cos 2\phi) - \sin^2\theta\sin 2\chi\sin 2\phi] \\
& - (1/2)\beta_{cab}[(\cos\theta - \cos^3\theta)(1 - \cos 2\chi)\sin 2\phi + \sin^2\theta\sin 2\chi(1 - \cos 2\phi)] \\
& + (1/8)\beta_{aab}[(\sin\theta - \sin^3\theta)(1 - \cos 2\chi)(\sin\phi + \sin 3\phi) + \sin\theta(1 + \cos 2\chi)(3\sin\phi - \sin 3\phi) \\
& \quad + 2\sin\theta\cos\theta \sin 2\chi(\cos\phi - \cos 3\phi)] \\
& + (1/8)\beta_{bbb}[(\sin\theta - \sin^3\theta)(1 - \cos 2\chi)(3\sin\phi - \sin 3\phi) + \sin\theta(1 + \cos 2\chi)(\sin\phi + \sin 3\phi) \\
& \quad - 2\sin\theta\cos\theta \sin 2\chi(\cos\phi - \cos 3\phi)] \\
& + (1/2)\beta_{cbb}\sin^3\theta(1 - \cos 2\chi)\sin\phi \\
& + (1/4)\beta_{abb}[(\sin\theta - \sin^3\theta)(1 - \cos 2\chi)(\cos\phi - \cos 3\phi) - \sin\theta(1 + \cos 2\chi)(\cos\phi - \cos 3\phi) \\
& \quad + 2\sin\theta\cos\theta \sin 2\chi(\sin\phi - \sin 3\phi)]
\end{aligned}$$

$$\begin{aligned}
(\text{psp}) \quad \chi_{XXY} = & (1/4)\beta_{cbb}\{[\sin\theta(\sin\chi - \sin 3\chi)(1 + \cos 2\phi) + (\sin\theta - \sin^3\theta)(\sin\chi + \sin 3\chi)(1 - \cos 2\phi)] \\
& \quad - 2\sin\theta\cos\theta\cos 3\chi\sin 2\phi\} \\
& - (1/4)\beta_{cab}\{ [2\sin\theta\sin\chi - (2\sin\theta - \sin^3\theta)(\sin\chi + \sin 3\chi)]\sin 2\phi \\
& \quad - 2\sin\theta\cos\theta(\cos\chi - \cos 3\chi)\cos 2\phi\} \\
& + (1/16)\beta_{aab}[(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)(\sin\phi + \sin 3\phi) + 4\cos\theta(\sin\chi\sin\phi - \sin 3\chi\sin 3\phi) \\
& \quad + \sin^2\theta(\cos\chi - \cos 3\chi)(\cos\phi - \cos 3\phi) - 4\cos^2\theta(\cos\chi\cos\phi - \cos 3\chi\cos 3\phi)] \\
& + (1/16)\beta_{bbb}[(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)(3\sin\phi - \sin 3\phi) - 4\cos\theta(\sin\chi\sin\phi - \sin 3\chi\sin 3\phi) \\
& \quad - \sin^2\theta(\cos\chi + 3\cos 3\chi)(\cos\phi - \cos 3\phi) + 4(\cos\chi\cos\phi - \cos 3\chi\cos 3\phi)] \\
& - (1/4)\beta_{cab}[(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)\sin\phi + \sin^2\theta(\cos\chi - \cos 3\chi)\cos\phi]
\end{aligned}$$

$$\begin{aligned}
& + (1/8)\beta_{abb}[(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)(\cos\phi - \cos 3\phi) - 4\cos\theta(\sin\chi\cos\phi - \sin 3\chi\cos 3\phi) \\
& \quad - \sin^2\theta(\cos\chi - \cos 3\chi)(\sin\phi + \sin 3\phi) - 4(\cos\chi\sin\phi - \cos 3\chi\sin 3\phi)] \\
\chi_{ZYX} = & -(1/2)\beta_{cbb}[(\sin\theta - 2\sin^3\theta)\sin\chi(1 - \cos 2\phi) - \sin\theta\cos\theta\cos\chi\sin 2\phi] \\
& - (1/2)\beta_{cab}[(\sin\theta - 2\sin^3\theta)\sin\chi\sin 2\phi + \sin\theta\cos\theta\cos\chi(1 - \cos 2\phi)] \\
& - (1/4)\beta_{aab}[(\cos\theta - \cos^3\theta)\sin\chi(\sin\phi + \sin 3\phi) + \sin^2\theta\cos\chi(3\cos\phi + \cos 3\phi)] \\
& + (1/4)\beta_{bbb}[-(\cos\theta - \cos^3\theta)\sin\chi(3\sin\phi - \sin 3\phi) + \sin^2\theta\cos\chi(\cos\phi - \cos 3\phi)] \\
& + \beta_{ccb}(\cos\theta - \cos^3\theta)\sin\chi\sin\phi \\
& - (1/2)\beta_{abb}[(\cos\theta - \cos^3\theta)\sin\chi(\cos\phi - \cos 3\phi) + \sin^2\theta\cos\chi(\sin\phi - \sin 3\phi)] \\
\chi_{XYZ} = & (1/2)\beta_{cbb}[(\cos\theta - \cos^3\theta)\sin 2\chi(1 - \cos 2\phi) - \sin^2\theta\cos 2\chi\sin 2\phi] \\
& + (1/2)\beta_{cab}[(\cos\theta - \cos^3\theta)\sin 2\chi\sin 2\phi + \sin^2\theta\cos 2\chi(1 - \cos 2\phi)] \\
& + (1/8)\beta_{aab}[-(\sin\theta - \sin^3\theta)\sin 2\chi(\sin\phi + \sin 3\phi) + \sin\theta\sin 2\chi(3\sin\phi - \sin 3\phi) \\
& \quad - 2\sin\theta\cos\theta\cos 2\chi(\cos\phi - \cos 3\phi)] \\
& + (1/8)\beta_{bbb}[-(\sin\theta - \sin^3\theta)\sin 2\chi(3\sin\phi - \sin 3\phi) + \sin\theta\sin 2\chi(\sin\phi + \sin 3\phi) \\
& \quad + 2\sin\theta\cos\theta\cos 2\chi(\cos\phi - \cos 3\phi)] \\
& - (1/2)\beta_{ccb}\sin^3\theta\sin 2\chi\sin\phi \\
& - (1/4)\beta_{abb}[(2\sin\theta - \sin^3\theta)\sin 2\chi(\cos\phi - \cos 3\phi) + 2\sin\theta\cos\theta\cos 2\chi(\sin\phi - \sin 3\phi)] \\
\chi_{ZYX} = & (1/4)\beta_{cbb}\{2[(\cos\theta - \cos^3\theta)(1 - \cos 2\phi) + \cos\theta\cos 2\phi]\sin 2\chi + [\sin^2\theta - (1 - 3\cos^2\theta)\cos 2\chi]\sin 2\phi\} \\
& - (1/4)\beta_{cab}[2\cos^3\theta\sin 2\chi\sin 2\phi - \sin^2\theta(1 - \cos 2\chi)(1 + \cos 2\phi) + 2\cos^2\theta(1 - \cos 2\chi\cos 2\phi)] \\
& - (1/8)\beta_{aab}\{(2\sin\theta - \sin^3\theta)\sin 2\chi(\sin\phi + \sin 3\phi) + 2\sin\theta\cos\theta[2\cos 2\phi - \cos 2\chi(\cos\phi + \cos 3\phi)]\} \\
& + (1/8)\beta_{bbb}[-(\sin\theta - \sin^3\theta)\sin 2\chi(3\sin\phi - \sin 3\phi) + \sin\theta\sin 2\chi(\sin\phi + \sin 3\phi) \\
& \quad + 2\sin\theta\cos\theta\cos 2\chi(\cos\phi - \cos 3\phi)] \\
& + (1/2)\beta_{ccb}[(\sin\theta - \sin^3\theta)\sin 2\chi\sin\phi + \sin\theta\cos\theta(1 - \cos 2\chi)\cos\phi] \\
& + (1/4)\beta_{abb}[-(\sin\theta - \sin^3\theta)\sin 2\chi(\cos\phi - \cos 3\phi) + \sin\theta\sin 2\chi(\cos\phi + \cos 3\phi) \\
& \quad - 2\sin\theta\cos\theta(\sin\phi + \cos 2\chi\sin 3\phi)]
\end{aligned}$$

$$\begin{aligned}
(\text{sps}) \quad \chi_{YXY} = & -(1/4)\beta_{cbb}\{[\sin\theta(\cos\chi + \cos 3\chi)(1 + \cos 2\phi) + (\sin\theta - \sin^3\theta)(\cos\chi - \cos 3\chi)(1 - \cos 2\phi)] \\
& \quad - 2\sin\theta\cos\theta\sin 3\chi\sin 2\phi\} \\
& + (1/4)\beta_{cab}\{[2\sin\theta\cos\chi - (2\sin\theta - \sin^3\theta)(\cos\chi - \cos 3\chi)]\sin 2\phi + 2\sin\theta\cos\theta(\sin\chi + \sin 3\chi\cos 2\phi)\} \\
& + (1/16)\beta_{aab}[-(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)(\sin\phi + \sin 3\phi) - 4\cos\theta(\cos\chi\sin\phi + \cos 3\chi\sin 3\phi) \\
& \quad + \sin^2\theta(\sin\chi + \sin 3\chi)(\cos\phi - \cos 3\phi) - 4\cos^2\theta(\sin\chi\cos\phi + \sin 3\chi\cos 3\phi)] \\
& + (1/16)\beta_{bbb}[-(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)(3\sin\phi - \sin 3\phi) + 4\cos\theta(\cos\chi\sin\phi + \cos 3\chi\sin 3\phi) \\
& \quad - \sin^2\theta(\sin\chi - 3\sin 3\chi)(\cos\phi - \cos 3\phi) + 4(\sin\chi\cos\phi + \sin 3\chi\cos 3\phi)] \\
& + (1/4)\beta_{ccb}[(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)\sin\phi - \sin^2\theta(\sin\chi + \sin 3\chi)\cos\phi] \\
& + (1/8)\beta_{abb}[-(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)(\cos\phi - \cos 3\phi) + 4\cos\theta(\cos\chi\cos\phi + \cos 3\chi\cos 3\phi) \\
& \quad - \sin^2\theta(\sin\chi + \sin 3\chi)(\sin\phi + \sin 3\phi) - 4\cos^2\theta(\sin\chi\sin\phi + \sin 3\chi\sin 3\phi)] \\
\chi_{YZY} = & (1/4)\beta_{cbb}\{2[\cos\theta(1 + \cos 2\phi\cos 2\chi) - (\cos\theta - \cos^3\theta)(1 - \cos 2\chi)(1 - \cos 2\phi)] \\
& \quad + (1 - 3\cos^2\theta)\sin 2\chi\sin 2\phi\} \\
& - (1/4)\beta_{cab}\{2[(\cos\theta - \cos^3\theta) + \cos^3\theta\cos 2\chi]\sin 2\phi - [\sin^2\theta + (1 - 3\cos^2\theta)\cos 2\phi]\sin 2\chi\} \\
& - (1/8)\beta_{aab}[(\sin\theta - \sin^3\theta)(1 - \cos 2\chi)(\sin\phi + \sin 3\phi) + \sin\theta(1 + \cos 2\chi)(\sin\phi + \sin 3\phi) \\
& \quad + 2\sin\theta\cos\theta\sin 2\chi(\cos\phi + \cos 3\phi)] \\
& + (1/8)\beta_{bbb}[(\sin\theta - \sin^3\theta)(1 - \cos 2\chi)(3\sin\phi - \sin 3\phi) + \sin\theta(1 + \cos 2\chi)(\sin\phi + \sin 3\phi)
\end{aligned}$$

$$\begin{aligned}
& - 2\sin\theta\cos\theta\sin 2\chi(\cos\phi - \cos 3\phi)] \\
& + (1/2)\beta_{c,cb}[-(\sin\theta - \sin^3\theta)(1 - \cos 2\chi)\sin\phi + \sin\theta\cos\theta\sin 2\chi\cos\phi] \\
& + (1/4)\beta_{a,bb}[(\sin\theta - \sin^3\theta)(1 - \cos 2\chi)(\cos\phi - \cos 3\phi) + \sin\theta(1 + \cos 2\chi)(\cos\phi + \cos 3\phi) \\
& \quad - 2\sin\theta\cos\theta\sin 2\chi\sin 3\phi]
\end{aligned}$$

(pps) $\chi_{XXY} = (1/4)\beta_{c,bb}\{[-\sin\theta(1 + \cos 2\phi) + (\sin\theta - \sin^3\theta)(1 - \cos 2\phi)](\sin\chi + \sin 3\chi) - 2\sin\theta\cos\theta(\cos\chi + \cos 3\chi)\sin 2\phi\}$

$$\begin{aligned}
& + (1/4)\beta_{c,cb}\{(2\sin\theta - \sin^3\theta)(\sin\chi + \sin 3\chi)\sin 2\phi - 2\sin\theta\cos\theta[2\cos\chi - (\cos\chi + \cos 3\chi)\cos 2\phi]\} \\
& + (1/16)\beta_{a,ab}[(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)(\sin\phi + \sin 3\phi) - 4\cos\theta(3\sin\chi\sin\phi + \sin 3\chi\sin 3\phi) \\
& \quad + \sin^2\theta(\cos\chi - \cos 3\chi)(\cos\phi - \cos 3\phi) + 4\cos^2\theta(3\cos\chi\cos\phi + \cos 3\chi\cos 3\phi)] \\
& + (1/16)\beta_{b,bb}[(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)(3\sin\phi - \sin 3\phi) - 4\cos\theta(\sin\chi\sin\phi - \sin 3\chi\sin 3\phi) \\
& \quad - \sin^2\theta(\cos\chi + 3\cos 3\chi)(\cos\phi - \cos 3\phi) + 4(\cos\chi\cos\phi - \cos 3\chi\cos 3\phi)] \\
& + (1/4)\beta_{c,cb}[-(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)\sin\phi + \sin^2\theta(3\cos\chi + \cos 3\chi)\cos\phi] \\
& + (1/8)\beta_{a,bb}[(\cos\theta - \cos^3\theta)(\sin\chi + \sin 3\chi)(\cos\phi - \cos 3\phi) + 4\cos\theta(\sin\chi\cos\phi + \sin 3\chi\cos 3\phi) \\
& \quad - \sin^2\theta(\cos\chi - \cos 3\chi)(\sin\phi + \sin 3\phi) + 4\cos^2\theta(\cos\chi\sin\phi + \cos 3\chi\sin 3\phi)]
\end{aligned}$$

$$\begin{aligned}
\chi_{ZZY} &= \beta_{c,bb}[-(\sin\theta - \sin^3\theta)\sin\chi(1 - \cos 2\phi) + \sin\theta\cos\theta\cos\chi\sin 2\phi] \\
& - \beta_{c,ab}[(\sin\theta - \sin^3\theta)\sin\chi\sin 2\phi - \sin\theta\cos\theta\cos\chi(1 + \cos 2\phi)] \\
& + (1/4)\beta_{a,ab}[-(\cos\theta - \cos^3\theta)\sin\chi(\sin\phi + \sin 3\phi) + \sin^2\theta\cos\chi(3\cos\phi + \cos 3\phi)] \\
& + (1/4)\beta_{b,bb}[-(\cos\theta - \cos^3\theta)\sin\chi(3\sin\phi - \sin 3\phi) + \sin^2\theta\cos\chi(\cos\phi - \cos 3\phi)] \\
& + \beta_{c,cb}(-\cos^3\theta\sin\chi\sin\phi + \cos^2\theta\cos\chi\cos\phi) \\
& + (1/2)\beta_{a,bb}[-(\cos\theta - \cos^3\theta)\sin\chi(\cos\phi - \cos 3\phi) + \sin^2\theta\cos\chi(\sin\phi + \sin 3\phi)] \\
\chi_{ZZY} &= (1/4)\beta_{c,bb}\{2[(\cos\theta - \cos^3\theta)(1 - \cos 2\phi) + \cos\theta\cos 2\phi]\sin 2\chi - [\sin^2\theta + (1 - 3\cos^2\theta)\cos 2\chi]\sin 2\phi\} \\
& - (1/4)\beta_{c,ab}[2\cos^3\theta\sin 2\chi\sin 2\phi + \sin^2\theta(1 + \cos 2\chi)(1 + \cos 2\phi) - 2\cos^2\theta(1 + \cos 2\chi\cos 2\phi)] \\
& + (1/8)\beta_{a,ab}\{-(2\sin\theta - \sin^3\theta)\sin 2\chi(\sin\phi + \sin 3\phi) + 2\sin\theta\cos\theta[2\cos\phi + \cos 2\chi(\cos\phi + \cos 3\phi)]\} \\
& + (1/8)\beta_{b,bb}\{-(\sin\theta - \sin^3\theta)\sin 2\chi(3\sin\phi - \sin 3\phi) + \sin\theta\sin 2\chi(\sin\phi + \sin 3\phi) \\
& \quad + 2\sin\theta\cos\theta\cos 2\chi(\cos\phi - \cos 3\phi)\} \\
& + (1/2)\beta_{c,cb}[(\sin\theta - \sin^3\theta)\sin 2\chi\sin\phi - \sin\theta\cos\theta(1 + \cos 2\chi)\cos\phi] \\
& + (1/4)\beta_{a,bb}[-(\sin\theta - \sin^3\theta)\sin 2\chi(\cos\phi - \cos 3\phi) + \sin\theta\sin 2\chi(\cos\phi + \cos 3\phi) \\
& \quad - 2\sin\theta\cos\theta\cos 2\chi(\sin\phi - \sin 3\phi)]
\end{aligned}$$

$$\begin{aligned}
\chi_{XZY} &= (1/4)\beta_{c,bb}\{2[(\cos\theta - \cos^3\theta)(1 - \cos 2\phi) + \cos\theta\cos 2\phi]\sin 2\chi - [\sin^2\theta + (1 - 3\cos^2\theta)\cos 2\chi]\sin 2\phi\} \\
& - (1/4)\beta_{c,ab}[2\cos^3\theta\sin 2\chi\sin 2\phi + \sin^2\theta(1 + \cos 2\chi)(1 + \cos 2\phi) - 2\cos^2\theta(1 + \cos 2\chi\cos 2\phi)] \\
& + (1/8)\beta_{a,ab}\{-(2\sin\theta - \sin^3\theta)\sin 2\chi(\sin\phi + \sin 3\phi) + 2\sin\theta\cos\theta[2\cos\phi + \cos 2\chi(\cos\phi + \cos 3\phi)]\} \\
& + (1/8)\beta_{b,bb}\{-(\sin\theta - \sin^3\theta)\sin 2\chi(3\sin\phi - \sin 3\phi) + \sin\theta\sin 2\chi(\sin\phi + \sin 3\phi) \\
& \quad + 2\sin\theta\cos\theta\cos 2\chi(\cos\phi - \cos 3\phi)\} \\
& + (1/2)\beta_{c,cb}[(\sin\theta - \sin^3\theta)\sin 2\chi\sin\phi - \sin\theta\cos\theta(1 + \cos 2\chi)\cos\phi] \\
& + (1/4)\beta_{a,bb}[-(\sin\theta - \sin^3\theta)\sin 2\chi(\cos\phi - \cos 3\phi) + \sin\theta\sin 2\chi(\cos\phi + \cos 3\phi) \\
& \quad - 2\sin\theta\cos\theta\cos 2\chi(\sin\phi - \sin 3\phi)]
\end{aligned}$$

(pss) $\chi_{XXY} = -(1/4)\beta_{c,bb}\{[\sin\theta(\cos\chi + \cos 3\chi)(1 + \cos 2\phi) + (\sin\theta - \sin^3\theta)(\cos\chi - \cos 3\chi)(1 - \cos 2\phi)] - 2\sin\theta\cos\theta\sin 3\chi\sin 2\phi\}$

$$\begin{aligned}
& + (1/4)\beta_{c,ab}\{[2\sin\theta\cos\chi - (2\sin\theta - \sin^3\theta)(\cos\chi - \cos 3\chi)]\sin 2\phi + 2\sin\theta\cos\theta(\sin\chi + \sin 3\chi\cos 2\phi)\}
\end{aligned}$$

$$\begin{aligned}
& - (1/16)\beta_{aab}[(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)(\sin\phi + \sin 3\phi) + 4\cos\theta(\cos\chi\sin\phi + \cos 3\chi\sin 3\phi) \\
& \quad - \sin^2\theta(\sin\chi + \sin 3\chi)(\cos\phi - \cos 3\phi) + 4\cos^2\theta(\sin\chi\cos\phi + \sin 3\chi\cos 3\phi)] \\
& - (1/16)\beta_{bbb}[(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)(3\sin\phi - \sin 3\phi) - 4\cos\theta(\cos\chi\sin\phi + \cos 3\chi\sin 3\phi) \\
& \quad + \sin^2\theta(\sin\chi - 3\sin 3\chi)(\cos\phi - \cos 3\phi) - 4\cos^2\theta(\sin\chi\cos\phi + \sin 3\chi\cos 3\phi)] \\
& + (1/4)\beta_{cab}[(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)\sin\phi - \sin^2\theta(\sin\chi + \sin 3\chi)\cos\phi] \\
& - (1/8)\beta_{abb}[(\cos\theta - \cos^3\theta)(\cos\chi - \cos 3\chi)(\cos\phi - \cos 3\phi) - 4\cos\theta(\cos\chi\cos\phi + \cos 3\chi\cos 3\phi) \\
& \quad + \sin^2\theta(\sin\chi + \sin 3\chi)(\sin\phi + \sin 3\phi) + 4\cos^2\theta(\sin\chi\sin\phi + \sin 3\chi\sin 3\phi)] \\
\chi_{ZYY} = & (1/4)\beta_{cbb}\{2[\cos\theta(1 + \cos 2\phi\cos 2\chi) - (\cos\theta - \cos^3\theta)(1 - \cos 2\chi)(1 - \cos 2\phi)] \\
& \quad + (1 - 3\cos^2\theta)\sin 2\chi\sin 2\phi\} \\
& - (1/4)\beta_{cab}\{2[(\cos\theta - \cos^3\theta) + \cos^3\theta\cos 2\chi]\sin 2\phi - [\sin^2\theta + (1 - 3\cos^2\theta)\cos 2\phi]\sin 2\chi\} \\
& + (1/8)\beta_{aab}[(\sin\theta - \sin^3\theta)(1 - \cos 2\chi)(\sin\phi + \sin 3\phi) - \sin\theta(1 + \cos 2\chi)(\sin\phi + \sin 3\phi) \\
& \quad - 2\sin\theta\cos\theta\sin 2\chi(\cos\phi + \cos 3\phi)] \\
& + (1/8)\beta_{bbb}[(\sin\theta - \sin^3\theta)(1 - \cos 2\chi)(3\sin\phi - \sin 3\phi) + \sin\theta(1 + \cos 2\chi)(\sin\phi + \sin 3\phi) \\
& \quad - 2\sin\theta\cos\theta\sin 2\chi(\cos\phi - \cos 3\phi)] \\
& + (1/2)\beta_{cab}[-(\sin\theta - \sin^3\theta)(1 - \cos 2\chi)\sin\phi + \sin\theta\cos\theta\sin 2\chi\cos\phi] \\
& + (1/4)\beta_{abb}[(\sin\theta - \sin^3\theta)(1 - \cos 2\chi)(\cos\phi - \cos 3\phi) + \sin\theta(1 + \cos 2\chi)(\cos\phi + \cos 3\phi) \\
& \quad - 2\sin\theta\cos\theta\sin 2\chi\sin 3\phi]
\end{aligned}$$

$$\begin{aligned}
(sss) \quad \chi_{YY} = & (1/4)\beta_{cbb}\{[\sin\theta(\sin\chi + \sin 3\chi)(1 + \cos 2\phi) + (\sin\theta - \sin^3\theta)(3\sin\chi - \sin 3\chi)(1 - \cos 2\phi)] \\
& \quad - 2\sin\theta\cos\theta(\cos\chi - \cos 3\chi)\sin 2\phi\} \\
& + (1/4)\beta_{cab}\{[4(\sin\theta - \sin^3\theta)\sin\chi - (2\sin\theta - \sin^3\theta)(\sin\chi + \sin 3\chi)]\sin 2\phi \\
& \quad - 2\sin\theta\cos\theta(\cos\chi - \cos 3\chi)\cos 2\phi\} \\
& + (1/16)\beta_{aab}[(\cos\theta - \cos^3\theta)(3\sin\chi - \sin 3\chi)(\sin\phi + \sin 3\phi) - 4\cos\theta(\sin\chi\sin\phi - \sin 3\chi\sin 3\phi) \\
& \quad - \sin^2\theta(\cos\chi - \cos 3\chi)(\cos\phi + 3\cos 3\phi) + 4(\cos\chi\cos\phi - \cos 3\chi\cos 3\phi)] \\
& + (1/16)\beta_{bbb}[(\cos\theta - \cos^3\theta)(3\sin\chi - \sin 3\chi)(3\sin\phi - \sin 3\phi) - 4\cos\theta(3\sin\chi\sin\phi + \sin 3\chi\sin 3\phi) \\
& \quad - 3\sin^2\theta(\cos\chi - \cos 3\chi)(\cos\phi - \cos 3\phi) + 4(3\cos\chi\cos\phi + \cos 3\chi\cos 3\phi)] \\
& + (1/4)\beta_{cab}[(\cos\theta - \cos^3\theta)(3\cos\chi + \cos 3\chi)\sin\phi + \sin^2\theta(\sin\chi + \sin 3\chi)\cos\phi] \\
& + (1/8)\beta_{abb}[(\cos\theta - \cos^3\theta)(3\sin\chi - \sin 3\chi)(\cos\phi - \cos 3\phi) - 4\cos\theta(\sin\chi\cos\phi + \sin 3\chi\cos 3\phi) \\
& \quad + \sin^2\theta(\cos\chi - \cos 3\chi)(\sin\phi - 3\sin 3\phi) - 4(\cos\chi\sin\phi + \cos 3\chi\sin 3\phi)]
\end{aligned}$$